UNIT 4
Electromagnetic Induction and Alternating Currents

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UNIT 4

Electromagnetic Induction and Alternating Currents

- In this unit, the student is exposed to
  - the phenomenon of electromagnetic induction
  - the application of Lenz's law to find the direction of induced emf
  - the concept of Eddy current and its uses
  - the phenomenon of self-induction and mutual-induction
  - the various methods of producing induced emfs
  - the construction and working of AC generators

- the principle of transformers and its role in long distance power communication
- the notion of root mean square value of alternating current
- the idea of phasors and phase relationships in different AC circuits
- the insight about power in an AC circuit and wattless current
- the understanding of energy conservation during LC oscillations

UNIT 4

Electromagnetic Induction and AC - Syllabus

- Electromagnetic Induction
  - Introduction
  - Magnetic flux
  - Faraday's laws of induction – Experiments
  - Lenz's law
  - Fleming's right hand rule
  - Motional emf from Lorentz force
  - Motional emf from Faraday's law and Energy conservation
- Eddy currents
  - Explanation
  - Applications of Eddy currents

- Self-induction
  - Introduction
  - Physical significance of inductance
  - Self-induction of a long solenoid
  - Mutual induction
  - Mutual Inductance of two long co-axial solenoids
- Methods of producing induced EMF
  - By changing magnetic induction
  - By changing area enclosed by the coil
  - By changing the orientation of the coil with field
- AC generator
  - Single-phase AC generator
  - Three-phase AC generator

- Transformer
    - Construction and working of Transformer
    - Energy losses in a transformer
    - Advantages of AC in long distance power transmission
- Alternating current
    - Mean or average value of AC
    - RMS value of AC
    - Phasor and phasor diagram
    - AC circuit with a resistor
    - AC circuit with an inductor
    - AC circuit with a capacitor
    - Series RLC circuit
    - Resonance in series RLC circuit

- Power in AC circuits
  - Introduction
  - Wattless current
  - Power factor
  - Advantages and disadvantages of AC over DC
- Oscillation in an LC circuit
  - Energy conversion during LC oscillations
  - Conservation of energy in LC oscillations
  - Analogies between LC oscillations and simple harmonic oscillations

4.1 Electromagnetic Induction

4.1.1 Introduction

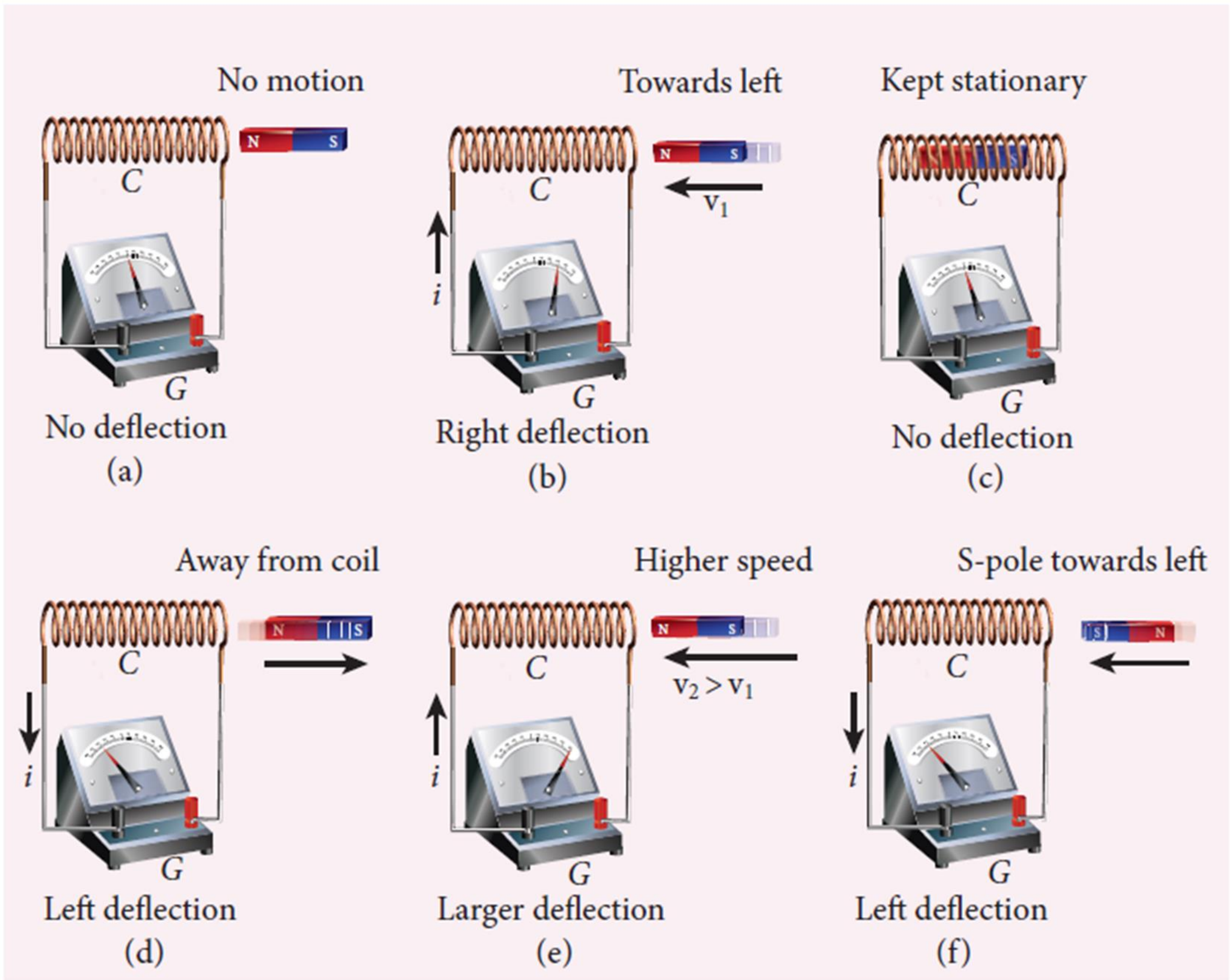
- Magnetic effects of electric current
- Its converse effect
- Is it possible to produce an electric current with the help of a magnetic field?

4.1.2 Magnetic flux

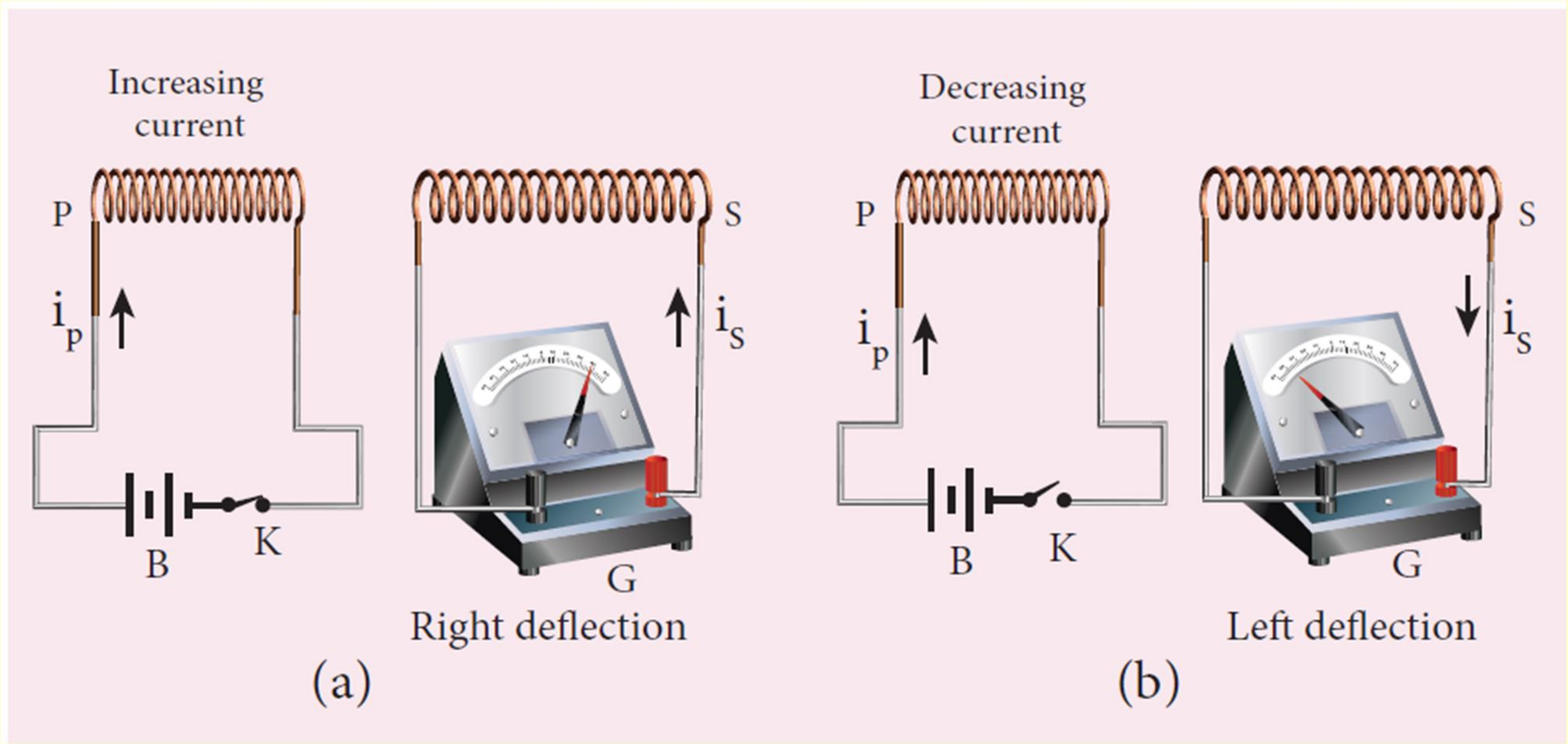
- the number of magnetic field lines passing through that area normally

$$\Phi_B = \int_A \vec{B}.\, d\vec{A} = BA \cos\theta \qquad (4.1)$$

4.1.3 Faraday's Experiments on Electromagnetic Induction - First Experiment



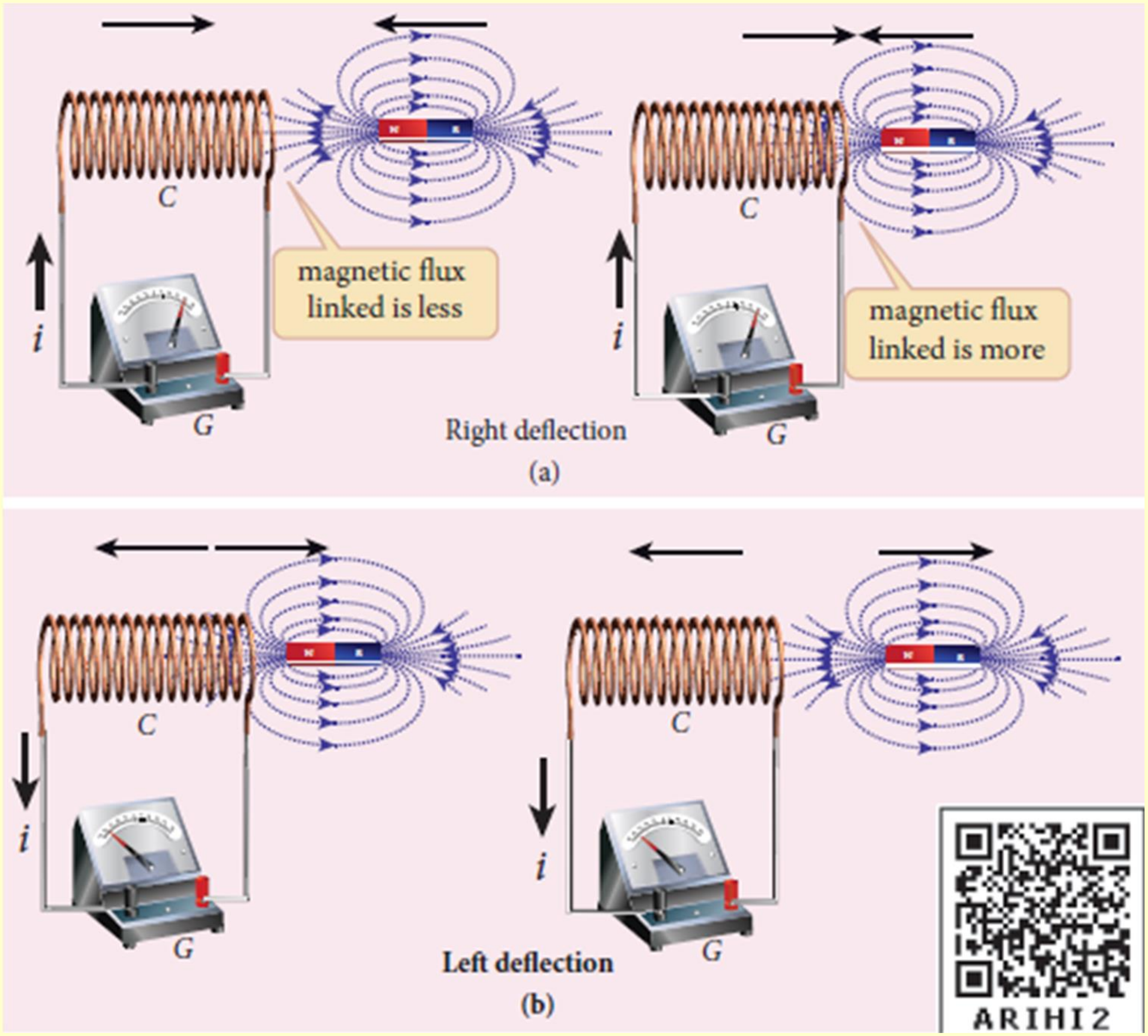
4.1.3 Faraday's Experiments on Electromagnetic Induction – Second Experiment



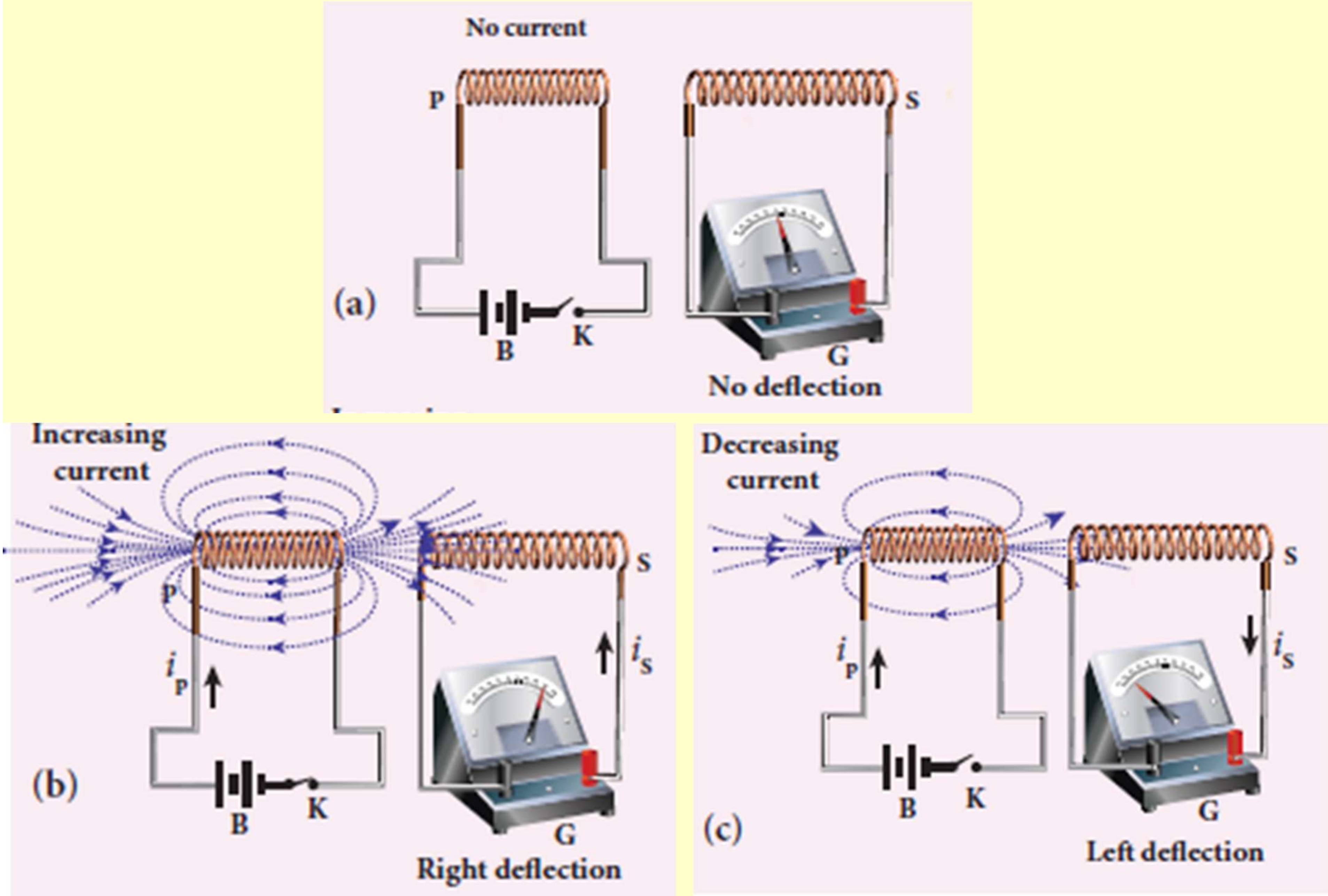
Electromagnetic Induction

- Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit.
- This current - an induced current and the emf - an induced emf.
- This is known as electromagnetic induction.

Explanation of Faraday's first experiment



Explanation of Faraday's second experiment



Faraday's laws of Electromagnetic induction

- First law

  Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

- Second law

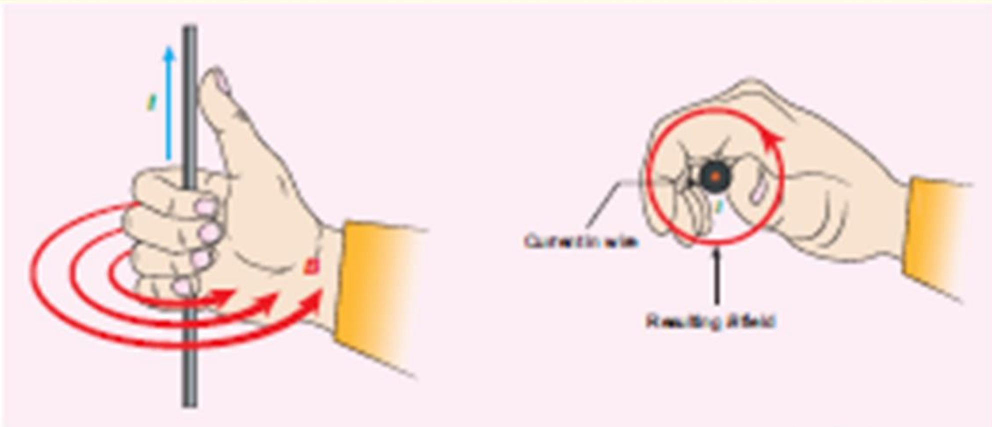
  The magnitude of induced emf in a closed circuit is equal to the time rate of change of magnetic flux linked with the circuit.

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -\frac{d(N\Phi_B)}{dt} \tag{4.3}$$

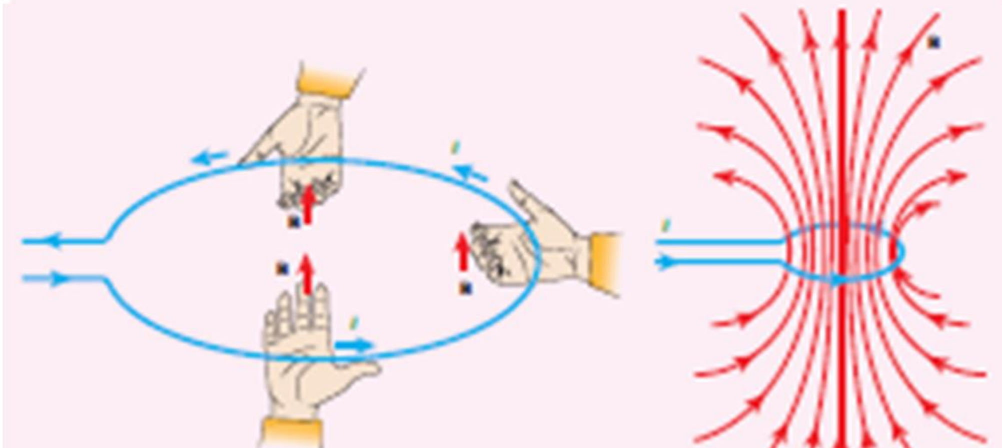
4.1.4 Lenz's law

- It states that the direction of the induced current is such that it always opposes the cause responsible for its production.
- This is according to the law of conservation of energy.
- It says that effect always opposes the cause.
- Causes:
  - In terms of increase or decrease of magnetic flux
  - In terms of movement of the magnets
- Effect:
  - Induction of emf or current
- The **direction of induced current** is found out.

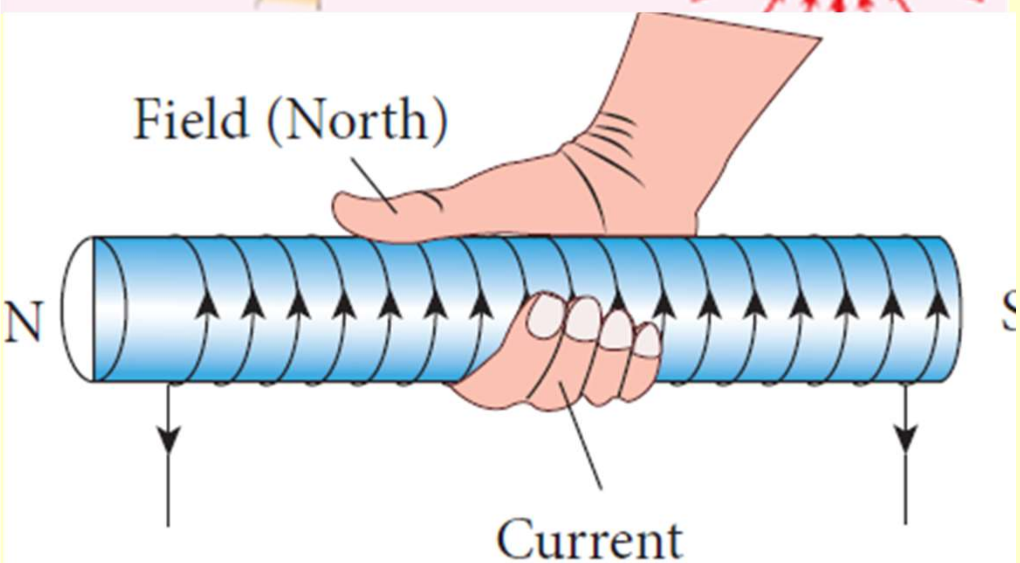
Right hand thumb rule



Direction of magnetic field of a straight conductor

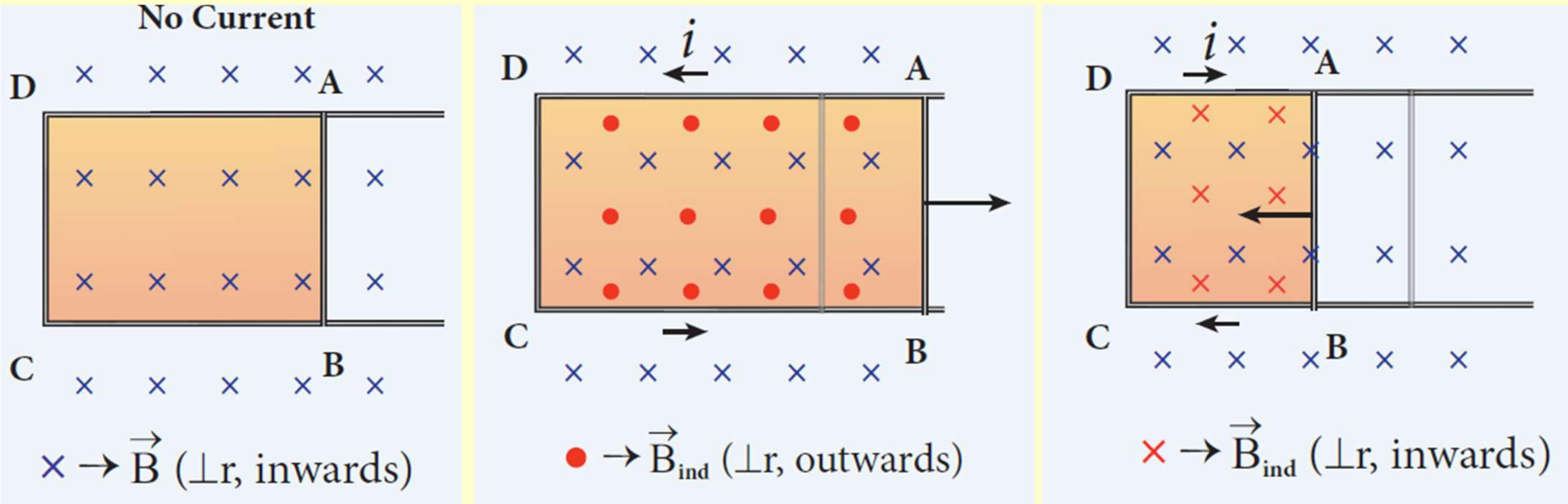


Direction of magnetic field of a circular loop



Direction of magnetic field of solenoid

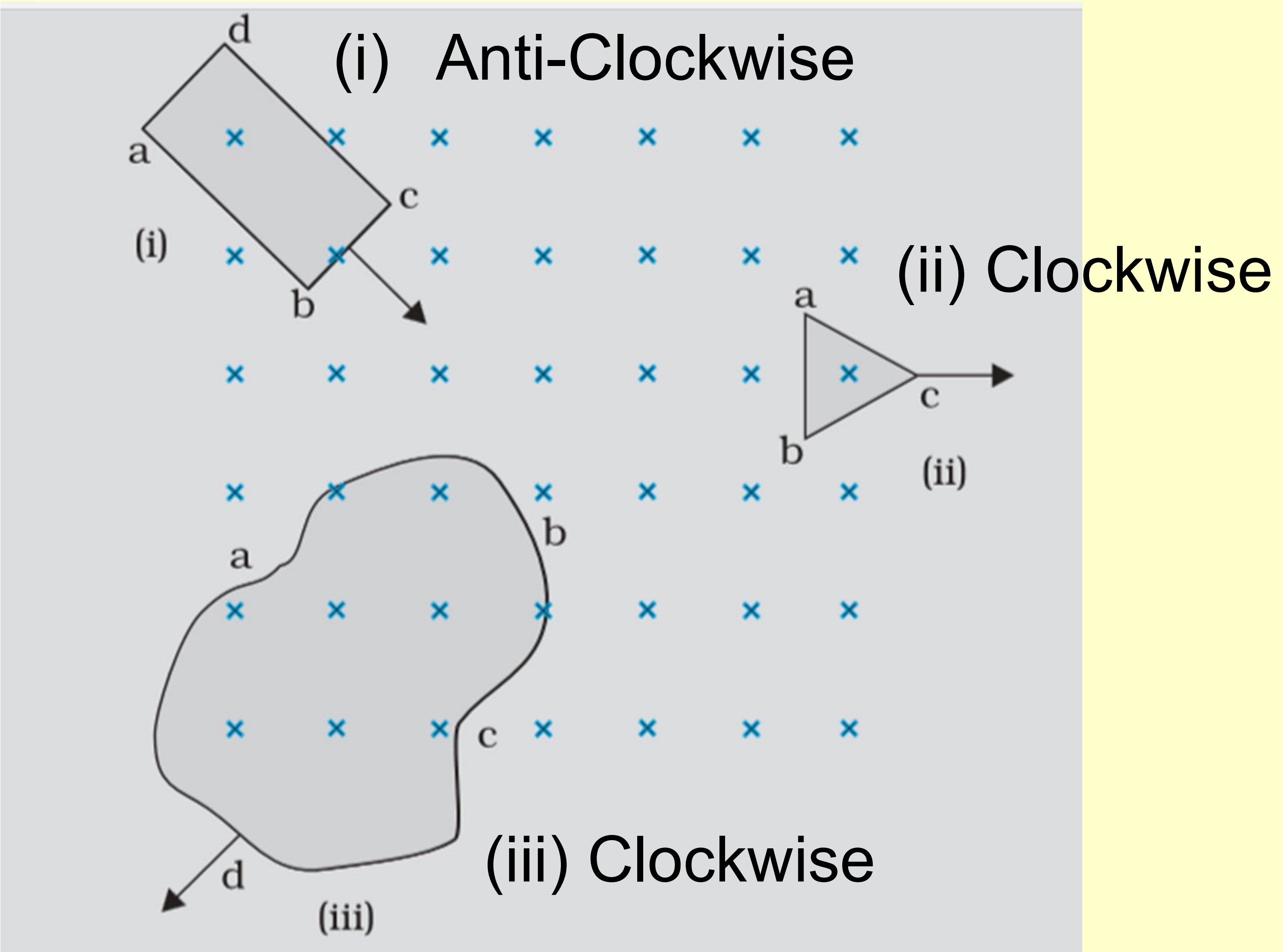
Lenz's law - First Illustration



If flux increases, opposite direction

If flux decreases, same direction

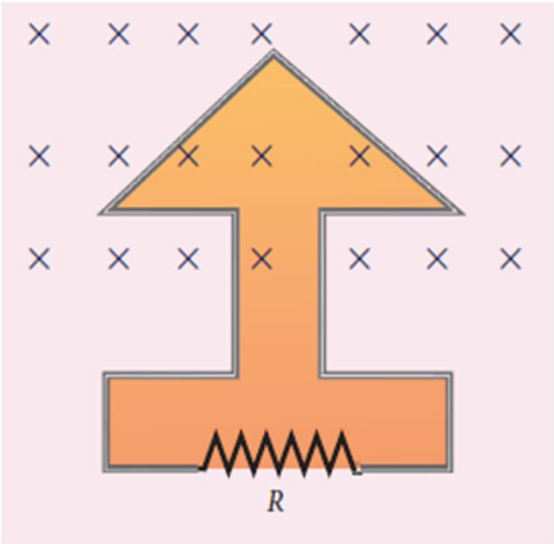
Find out the direction of the induced currents in the 3 closed loops given below using Lenz's law.



Lenz's law - First Illustration

EXAMPLE 4.7

The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = (2t^3 + 3t^2 + 8t + 5)\,mWb$, what is the magnitude of the induced emf in the loop when $t = 3$ s? Find out the direction of current through the circuit.



(i) $$\varepsilon = \frac{d(N\Phi_B)}{dt}$$

$$= \frac{d}{dt}(2t^3 + 3t^2 + 8t + 5) \times 10^{-3}$$

$$= (6t^2 + 6t + 8) \times 10^{-3}$$

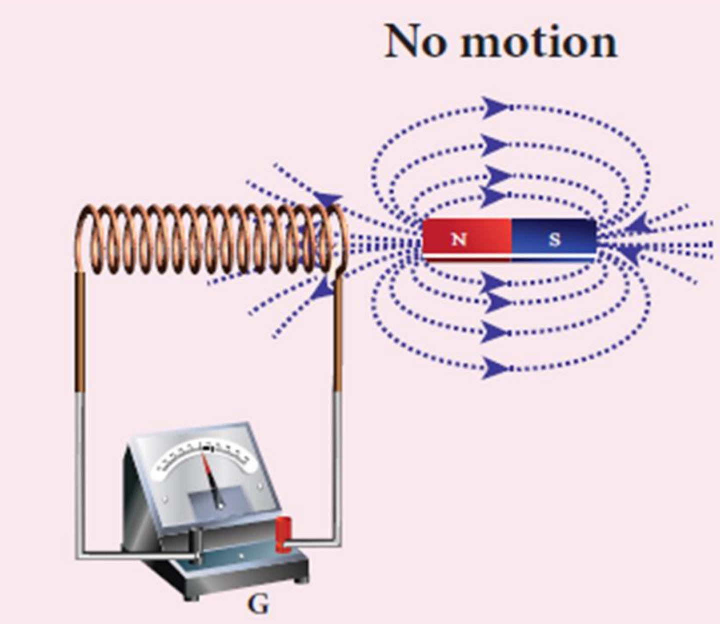
At t = 3 s,

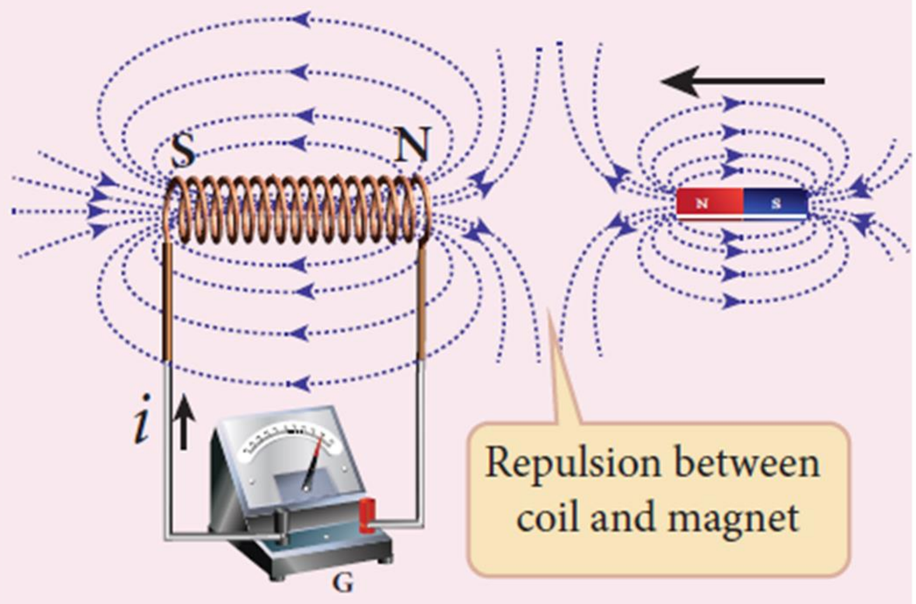
$$\varepsilon = [(6 \times 9) + (6 \times 3) + 8] \times 10^{-3}$$

$$= 80 \times 10^{-3} V \quad = 80\,mV$$

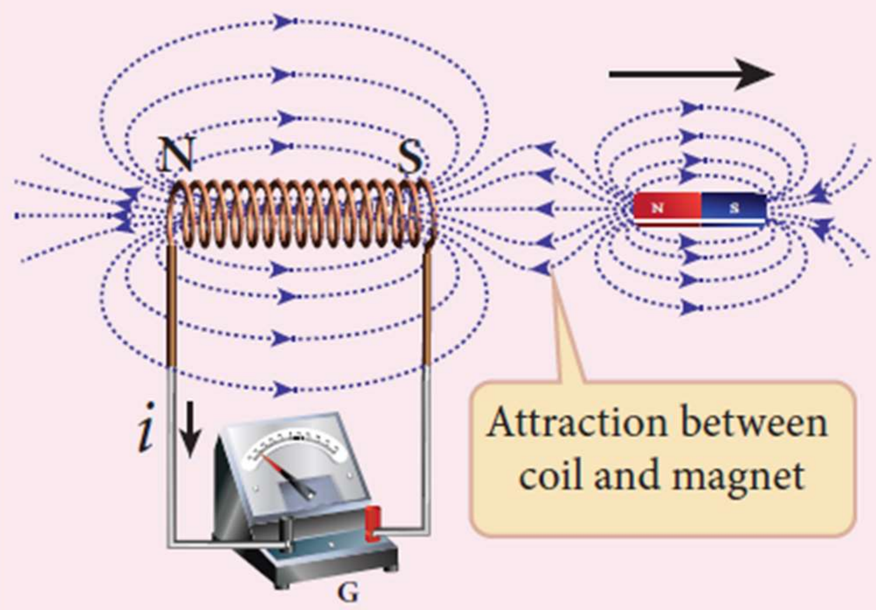
(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anti-clockwise direction.

Lenz's law - Second Illustration



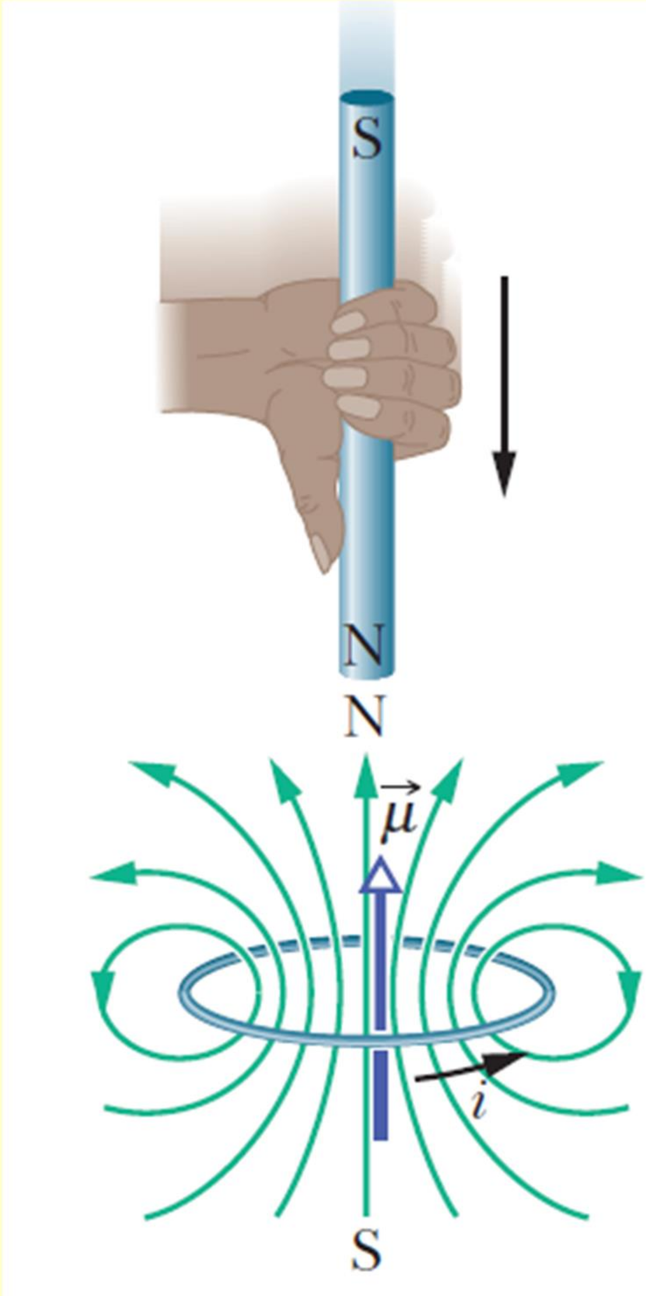


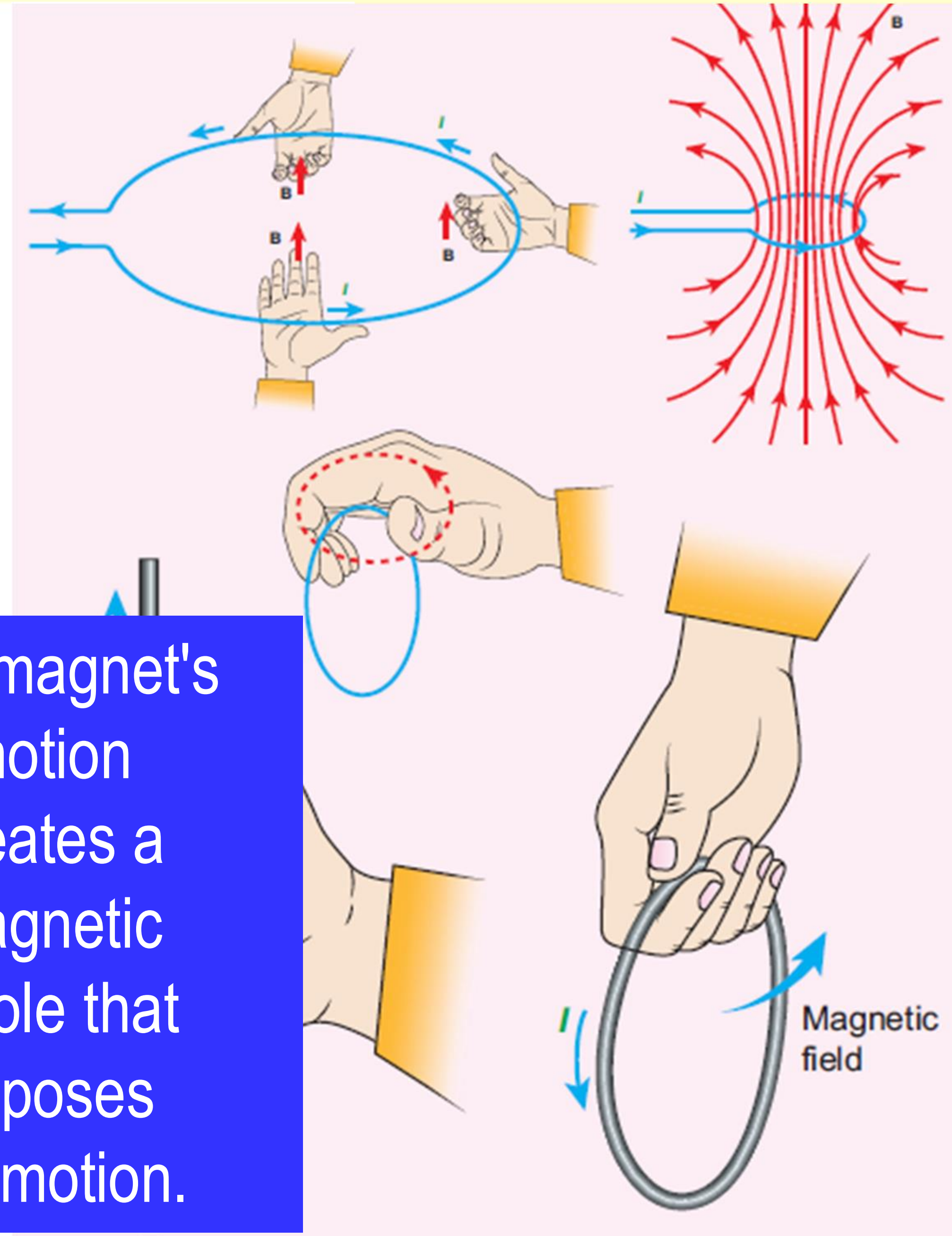
(b)



(c)

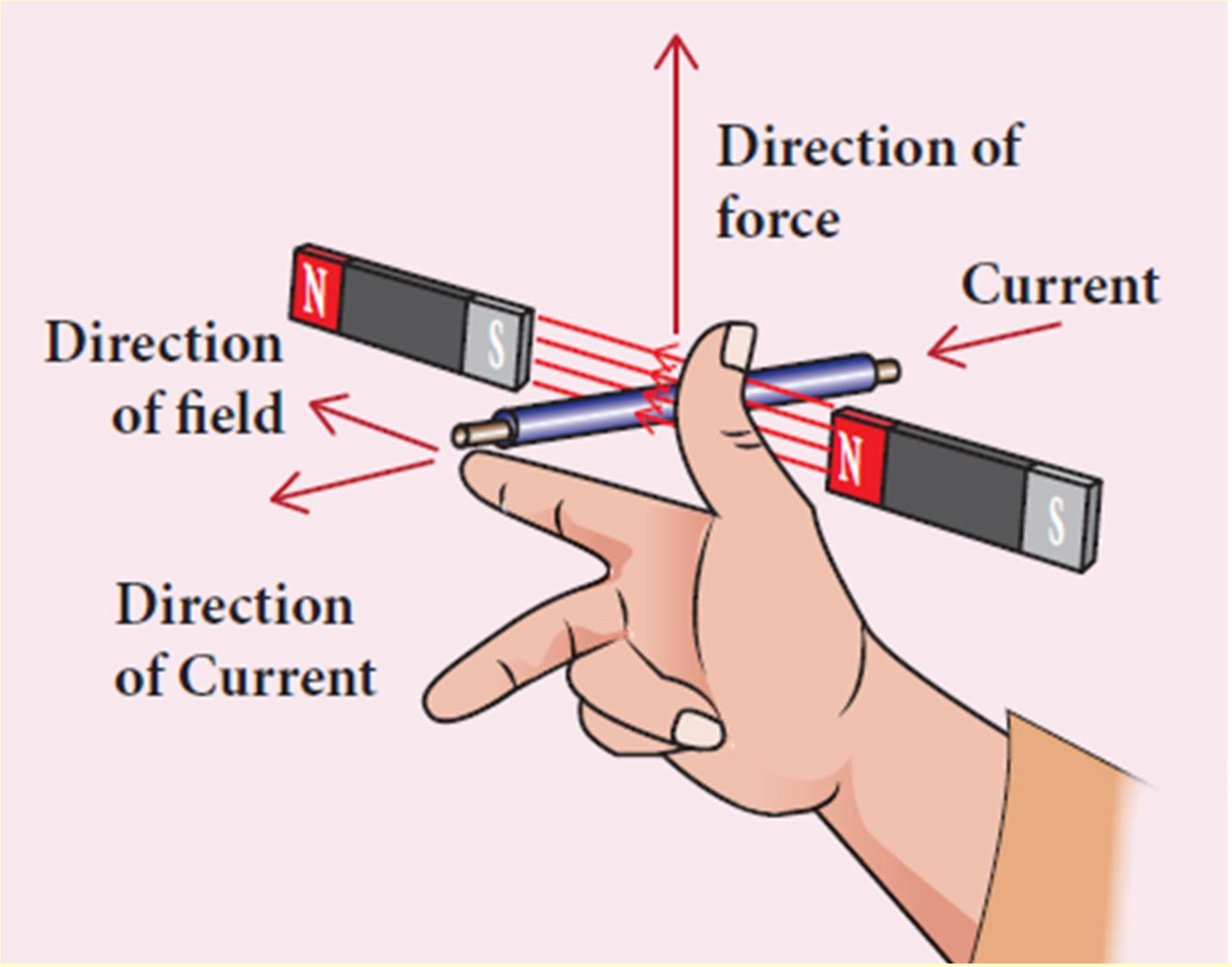
Lenz’s law - Second Illustration



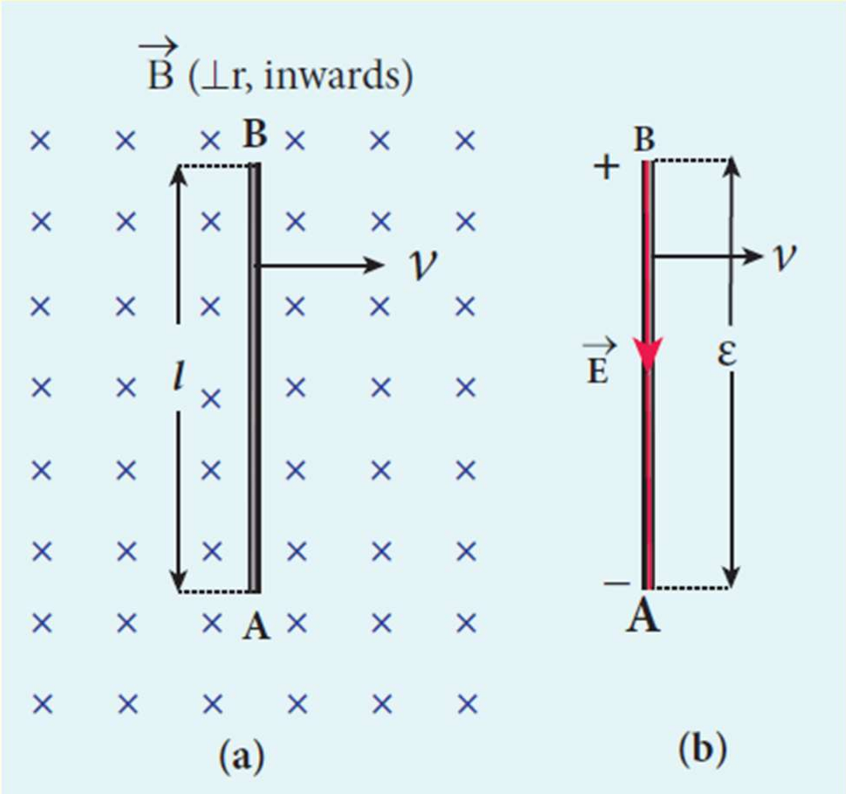


The magnet's motion creates a magnetic dipole that opposes the motion.

4.1.5 Fleming's right hand rule



4.1.6 Motional emf from Lorentz force



$$\overrightarrow{F_B} = -e\left(\vec{v} \times \vec{B}\right)$$

$$\overrightarrow{F_E} = -e\vec{E}$$

i.e.,
$$\left|\vec{F}_B\right| = \left|\vec{F}_E\right|$$

$$\left|-e\ (\vec{v} \times \vec{B})\right| = \left|-e\vec{E}\right|$$

$$vB\sin 90° = E$$

$$vB = E$$

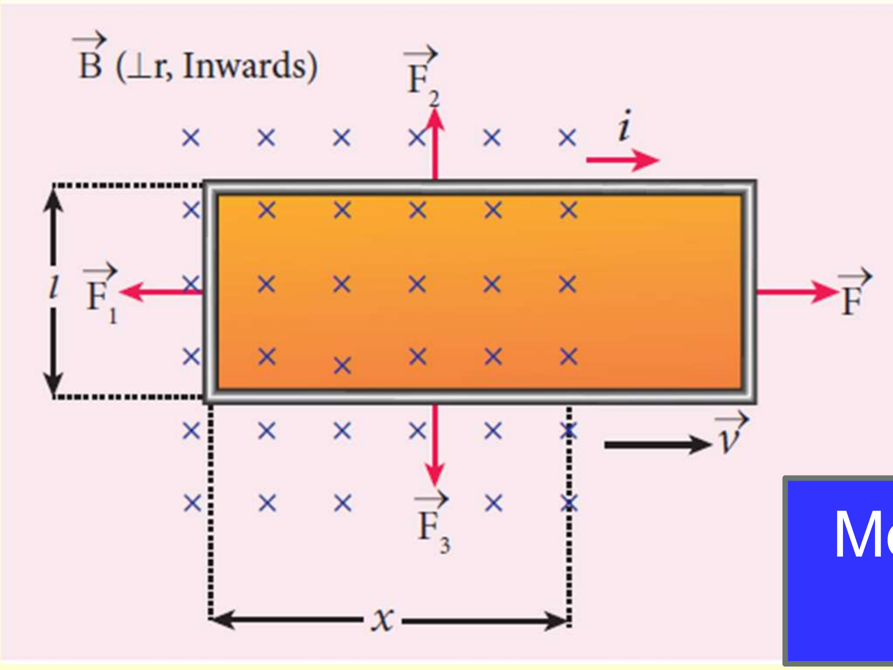
$$V = El$$

$$V = vBl$$

Motional emf is

$$\boxed{\varepsilon = Blv}$$

4.1.7 Motional emf from Faraday's law and Energy conservation



$$\Phi_B = \int_A \vec{B} \cdot d\vec{A} = BA\cos\theta$$

Here $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$$= BA$$

$$\Phi_B = Blx \qquad (4.8)$$

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(Blx)$$

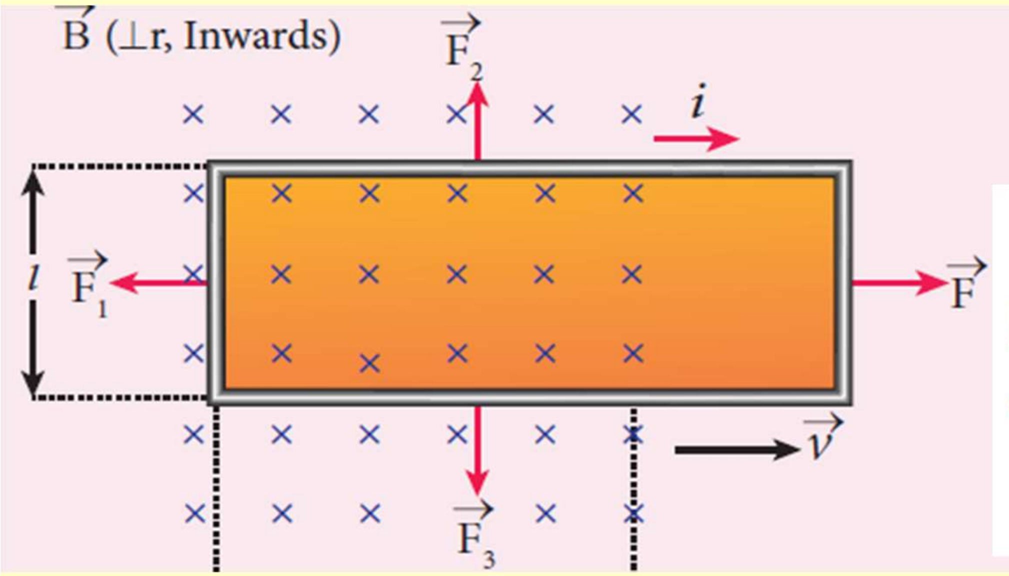
$$\varepsilon = Bl\frac{dx}{dt}$$

Motional emf

$$\boxed{\varepsilon = Blv}$$

$$i = \frac{\varepsilon}{R}$$

$$i = \frac{Blv}{R}$$



Energy conservation

The applied force $\vec{F}$ must be equal to $\vec{F}_1$ in order to just move the loop with a constant velocity $\vec{v}$

$$\therefore \vec{F} = -\vec{F}_1$$

$$F = F_1 = i\,l\,B$$

$$F = \left(\frac{Blv}{R}\right) lB$$

$$F = \frac{B^2 l^2 v}{R}$$

$$P = Fv = \left(\frac{B^2 l^2 v}{R}\right) v$$

$$P = \frac{B^2 l^2 v^2}{R}$$

The rate of doing work or power

$$P = i^2 R$$

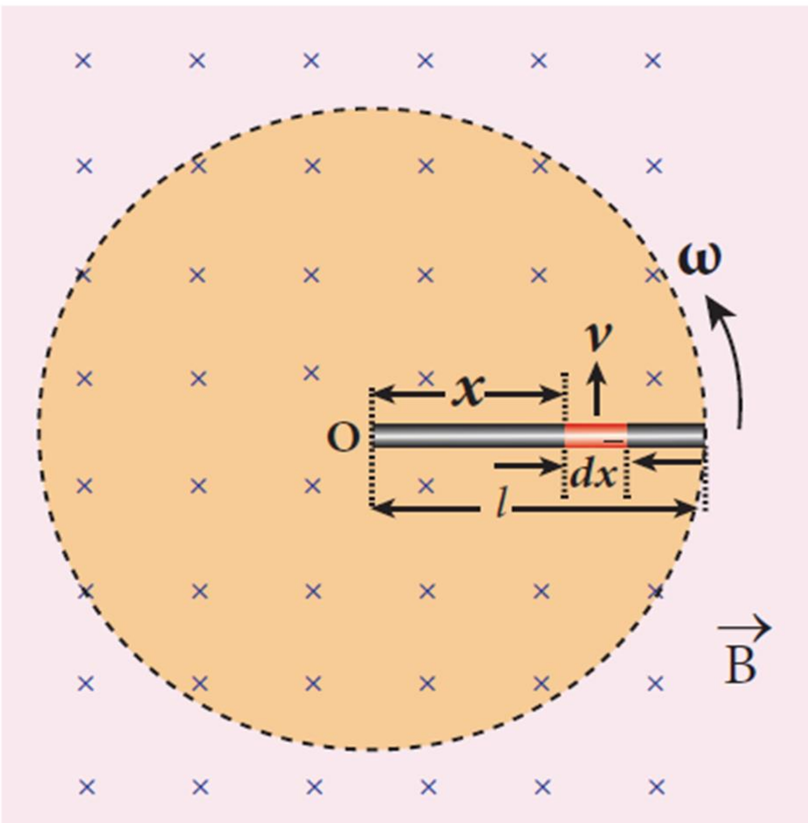
$$P = \left(\frac{Blv}{R}\right)^2 R$$

$$P = \frac{B^2 l^2 v^2}{R}$$

The rate at which thermal energy is dissipated or power

EXAMPLE 4.9

A copper rod of length l rotates about one of its ends with an angular velocity ω in a magnetic field B as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.



Solution

Consider a small element of length dx at a distance x from the centre of the circle described by the rod. As this element moves perpendicular to the field with a linear velocity $v = x\omega$, the emf developed in the element dx is

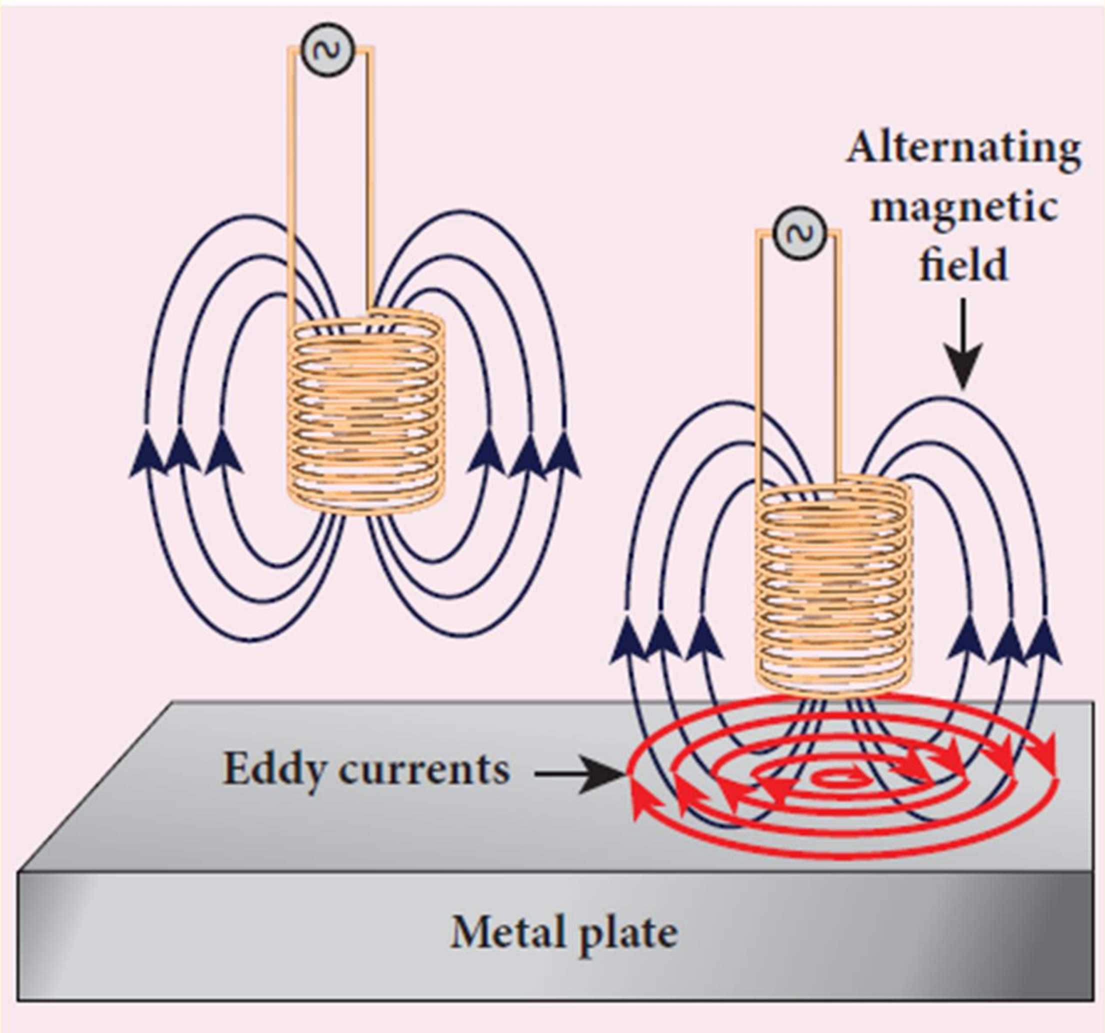
$$d\varepsilon = Bv dx \qquad = B(x\omega)dx$$

This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

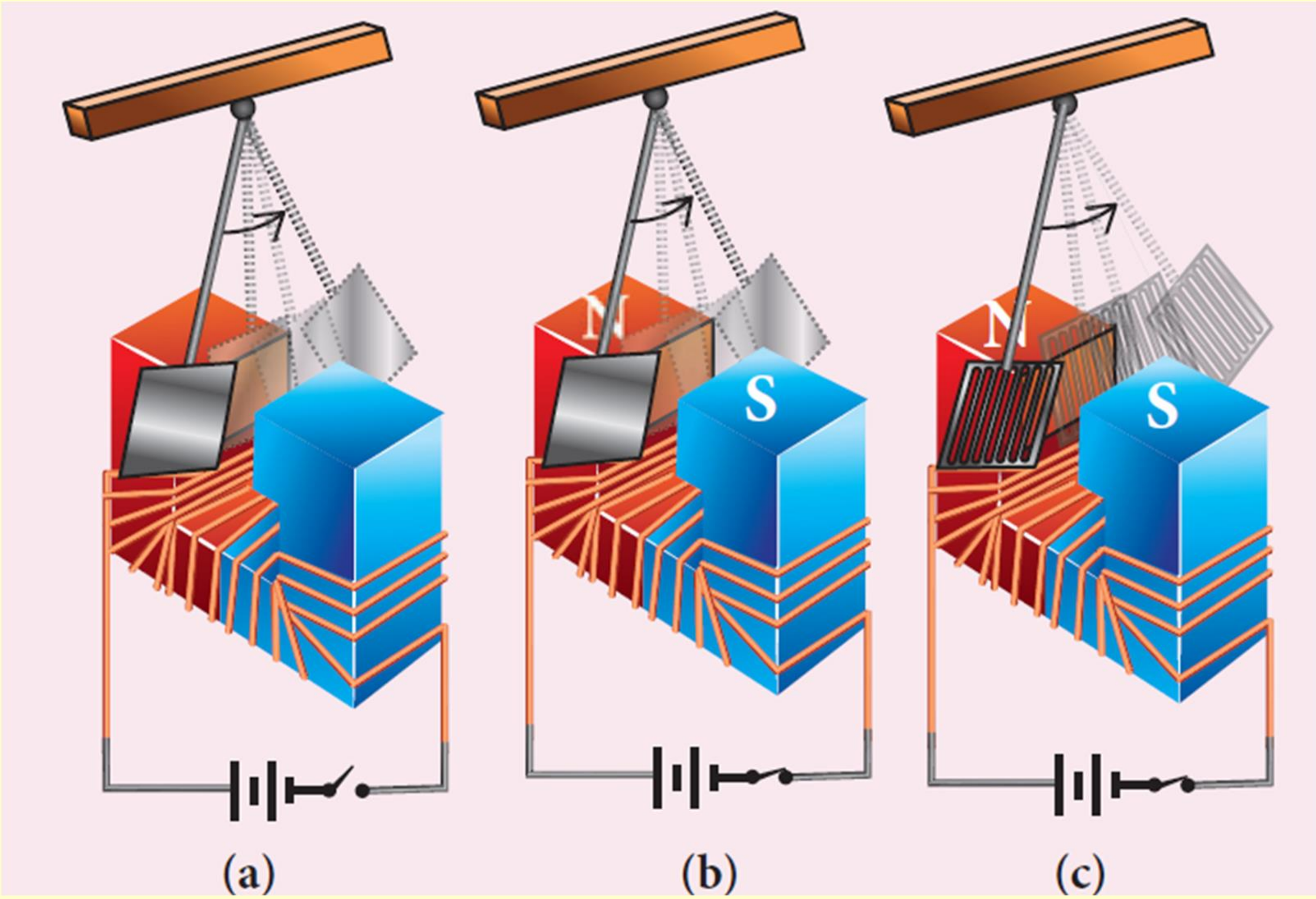
$$\varepsilon = \int d\varepsilon = \int_0^l B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_0^l$$

$$\varepsilon = \frac{1}{2}B\omega l^2$$

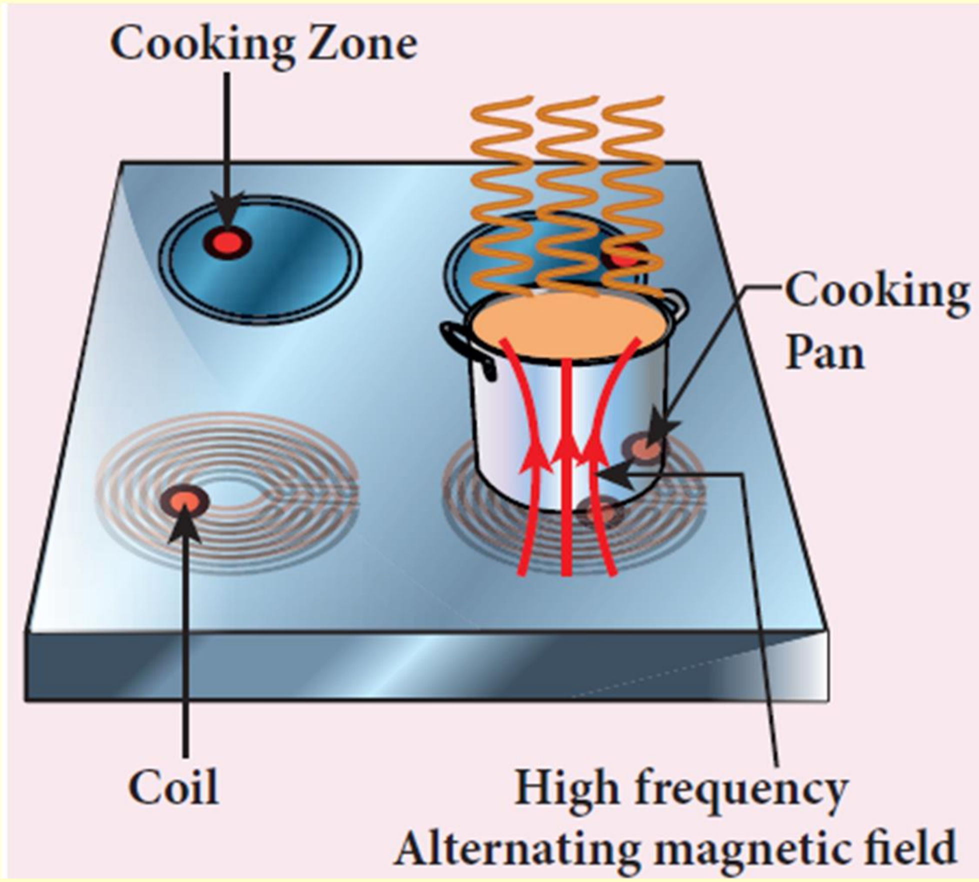
4.2 Eddy currents



Eddy currents - Demonstration



i. Induction stove



ii. Eddy current brake

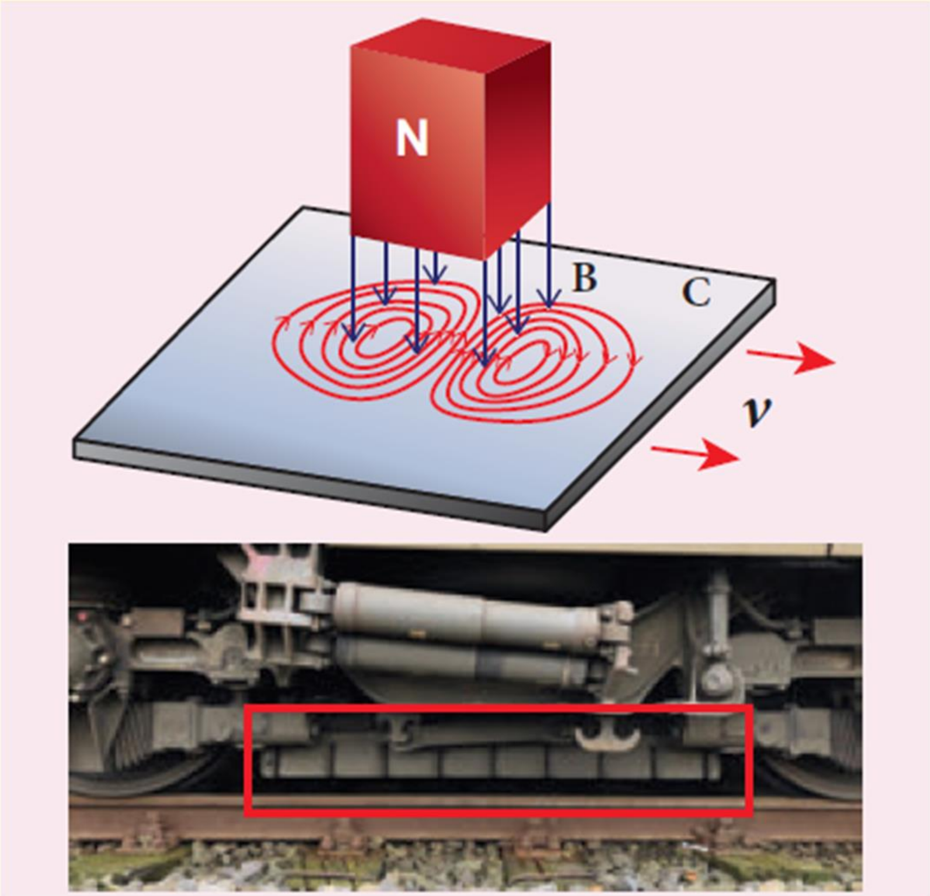


Figure 4.15 (a) Linear Eddy current brake

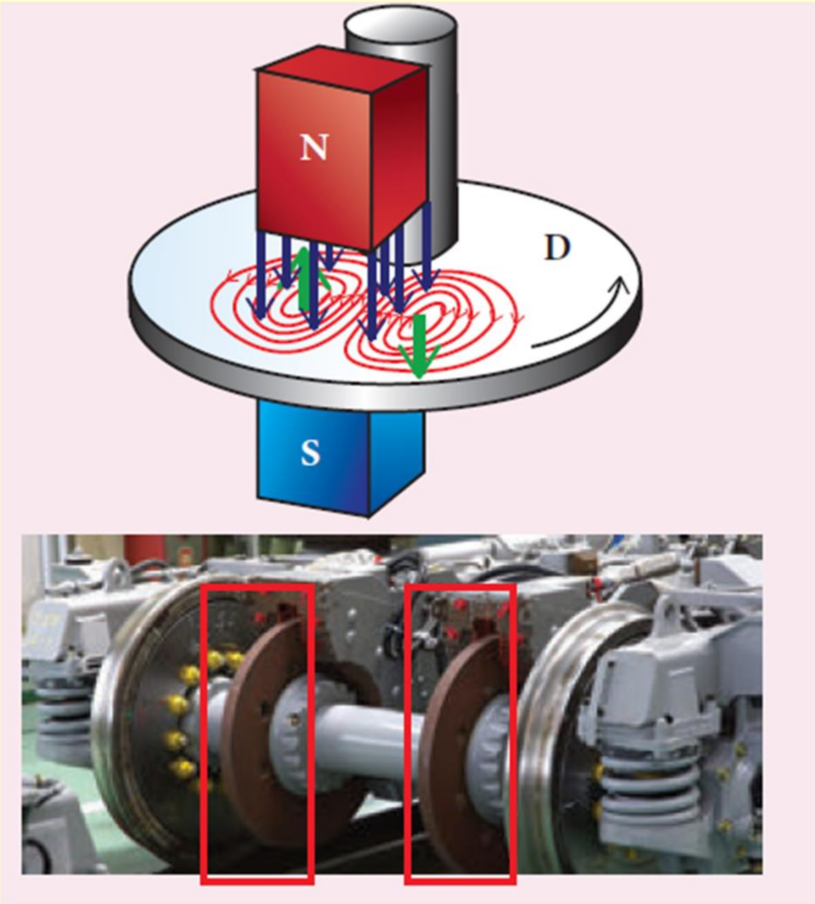


Figure 4.15(b) Circular Eddy current brake

iii. Eddy current testing

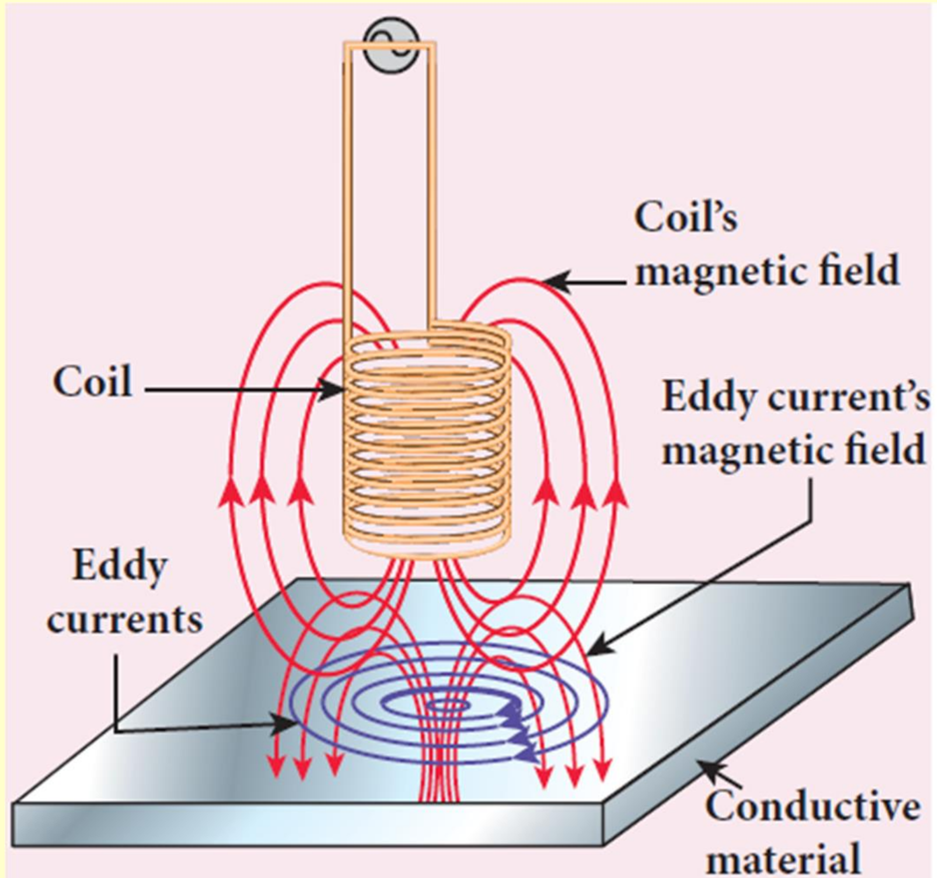


Figure 4.16 Eddy current testing

iv. Electromagnetic damping

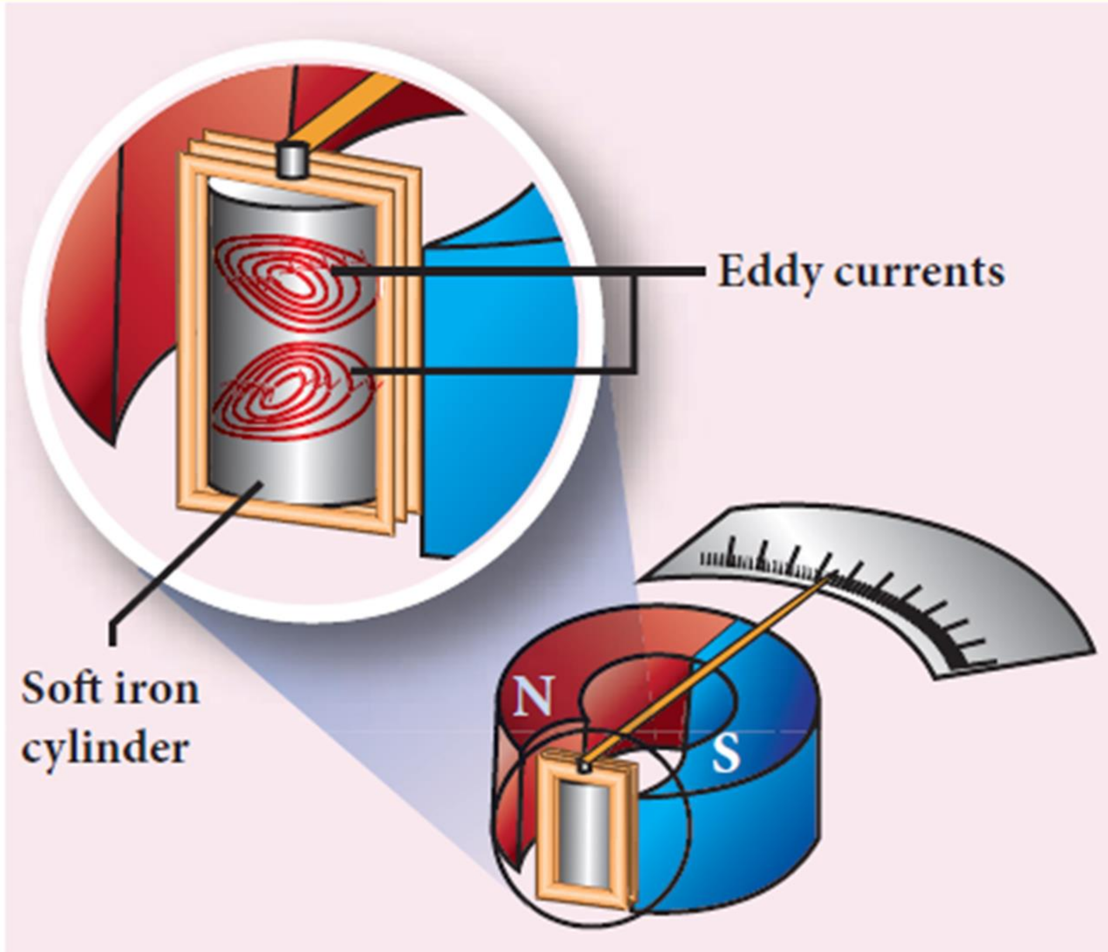


Figure 4.17 Electromagnetic damping

4.3 Self-induction

4.3.1 Introduction

- Inductor is a device used to store energy in a magnetic field when an electric current flows through it.

Self – induction

- If flux linked with the coil is changed by changing the current, an emf is induced in that same coil. This phenomenon is known as self-induction.
- The emf induced is called self-induced emf.

Physical significance

- L plays same role as mass and MI.
- Thus, inductance of the coil opposes any change in current and tries to maintain the original state.

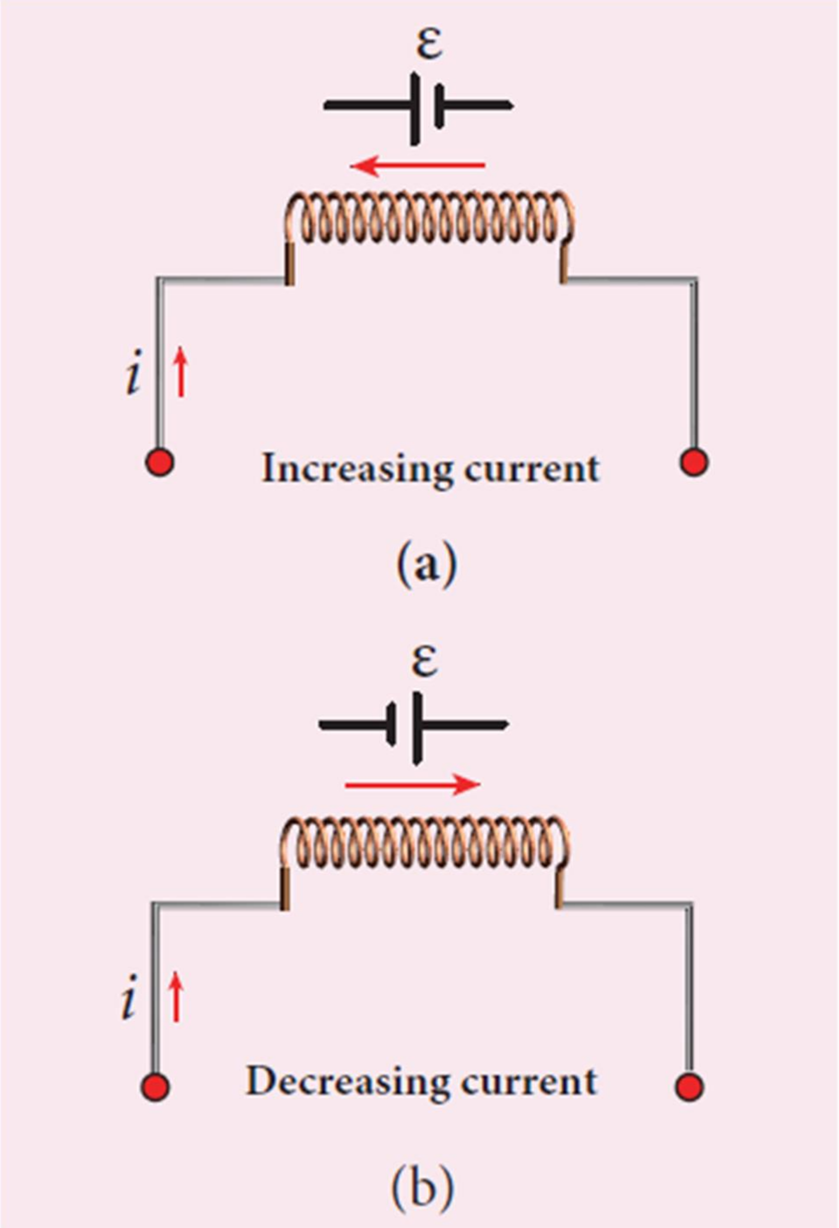
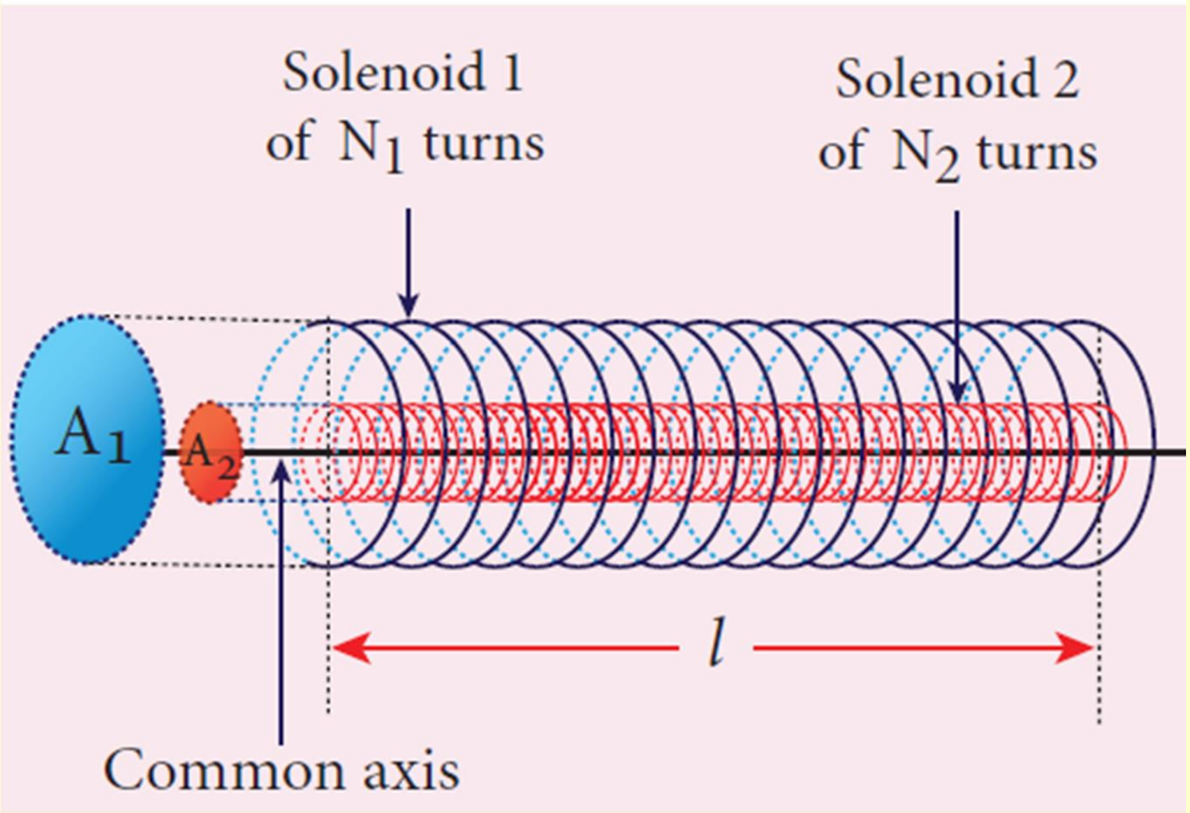


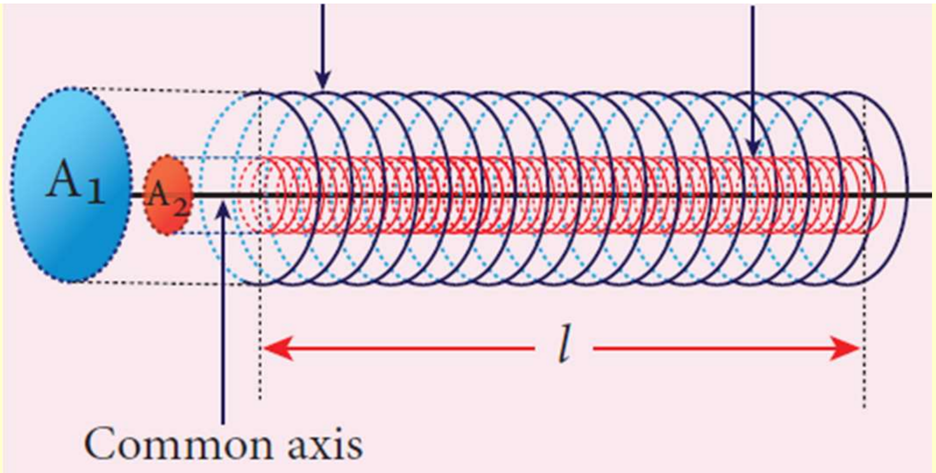
Figure 4.20 Induced emf ε opposes the changing current *i*

4.3.3 Mutual induction

- When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil.
- This is mutual induction
- Emf is mutually induced emf

4.3.4 Mutual inductance between two long co-axial solenoids





Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is

$$B_1 = \mu_\circ n_1 i_1$$

As the field lines of $\vec{B_1}$ are passing through the area bounded by solenoid 2, the magnetic flux is linked with each turn of solenoid 2 due to solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \vec{B_1}.d\vec{A} = B_1 A_2 \qquad \text{since } \theta = 0^\circ$$

$$= (\mu_\circ n_1 i_1) A_2$$

The flux linkage of solenoid 2 with total turns N_2 is

$$N_2\Phi_{21} = (n_2 l)(\mu_\circ n_1 i_1) A_2 \qquad \text{since } N_2 = n_2 l$$

$$N_2\Phi_{21} = (\mu_\circ n_1 n_2 A_2 l) i_1 \qquad (4.20)$$

From equation (4.19),

$$N_2\Phi_{21} = M_{21} i_1 \qquad (4.21)$$

Comparing the equations (4.20) and (4.21),

$$M_{21} = \mu_\circ n_1 n_2 A_2 l \qquad (4.22)$$

The magnetic field produced by the solenoid 2 when carrying a current i_2 is

$$B_2 = \mu_o n_2 i_2$$

This magnetic field B_2 is uniform inside the solenoid 2 but outside the solenoid 2, it is almost zero. Therefore for solenoid 1, the area A_2 is the effective area over which the magnetic field B_2 is present; not area A_1. Then the magnetic flux Φ_{12} linked with each turn of solenoid 1 due to solenoid 2 is

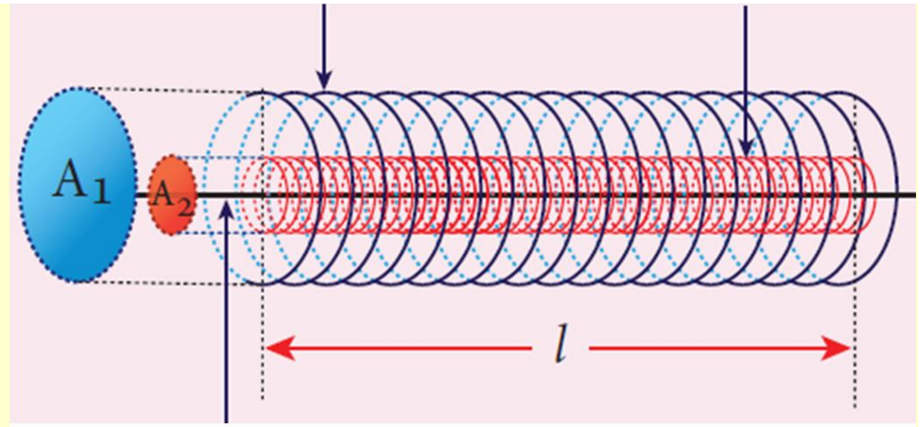
$$\Phi_{12} = \int_{A_2} \vec{B}_2 \cdot d\vec{A} = B_2 A_2 = (\mu_o n_2 i_2) A_2$$

The flux linkage of solenoid 1 with total turns N_1 is

$$N_1 \Phi_{12} = (n_1 l)(\mu_o n_2 i_2) A_2 \qquad \text{since } N_1 = n_1 l$$

$$N_1 \Phi_{12} = (\mu_o n_1 n_2 A_2 l) i_2$$

$$\text{since } N_1 \Phi_{12} = M_{12} i_2$$



$$M_{12} i_2 = (\mu_o n_1 n_2 A_2 l) i_2$$

Therefore, we get

$$\therefore M_{12} = \mu_o n_1 n_2 A_2 l \qquad (4.23)$$

From equation (4.22) and (4.23), we can write

$$M_{12} = M_{21} = M \qquad (4.24)$$

In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_o n_1 n_2 A_2 l \qquad (4.25)$$

4.4. Methods of producing induced emf

Induced emf can be produced by changing magnetic flux in any of the following ways.

- By changing the magnetic field B
- By changing the area A of the coil and
- By changing the relative orientation θ of the coil with magnetic field

4.4.4 Induction of emf by changing relative orientation of the coil with the magnetic field

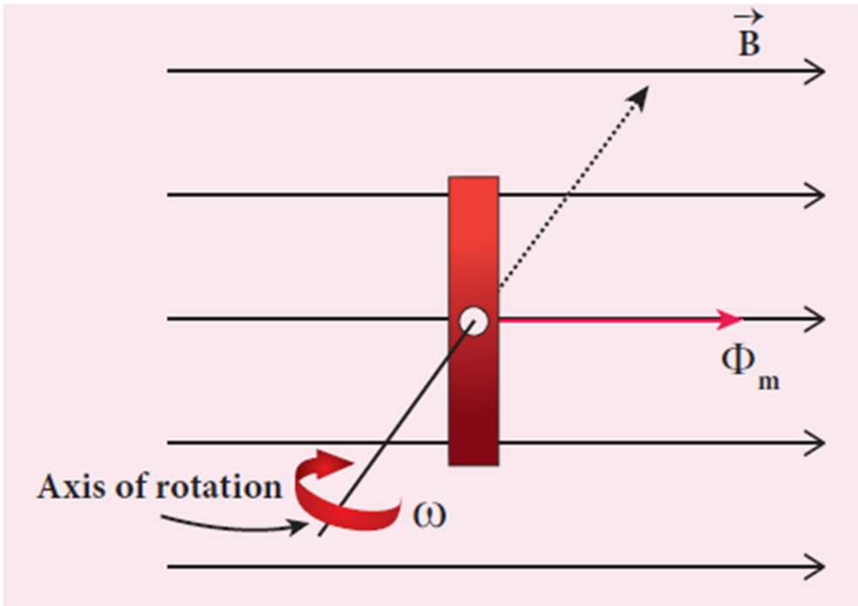


Figure 4.25(a) Top view of the coil with its plane perpendicular to the magnetic field

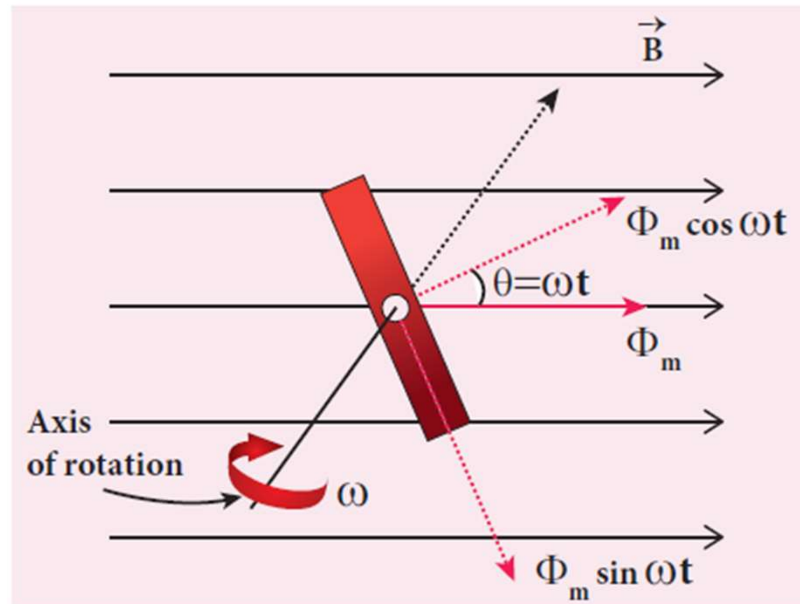


Figure 4.25(b) The coil has rotated through an angle $\theta = \omega t$

$$\Phi_B = \Phi_m \xrightarrow{\text{FLUX CHANGE}} \Phi_B = \Phi_m \cos\omega t$$

$$\varepsilon = -\frac{d}{dt}(N\Phi_B) = -\frac{d}{dt}(N\Phi_m \cos\omega t)$$

$$= -N\Phi_m(-\sin\omega t)\omega$$

$$= N\Phi_m \omega \sin\omega t$$

When the coil is rotated through 90° from initial position, $\sin\omega t = 1$. Then the maximum value of induced emf is

$$\varepsilon_m = N\Phi_m\omega$$

$$\varepsilon_m = NBA\omega \qquad \text{since } \Phi_m = BA$$

Therefore, the value of induced emf at that instant is then given by

$$\varepsilon = \varepsilon_m \sin\omega t \qquad (4.28)$$

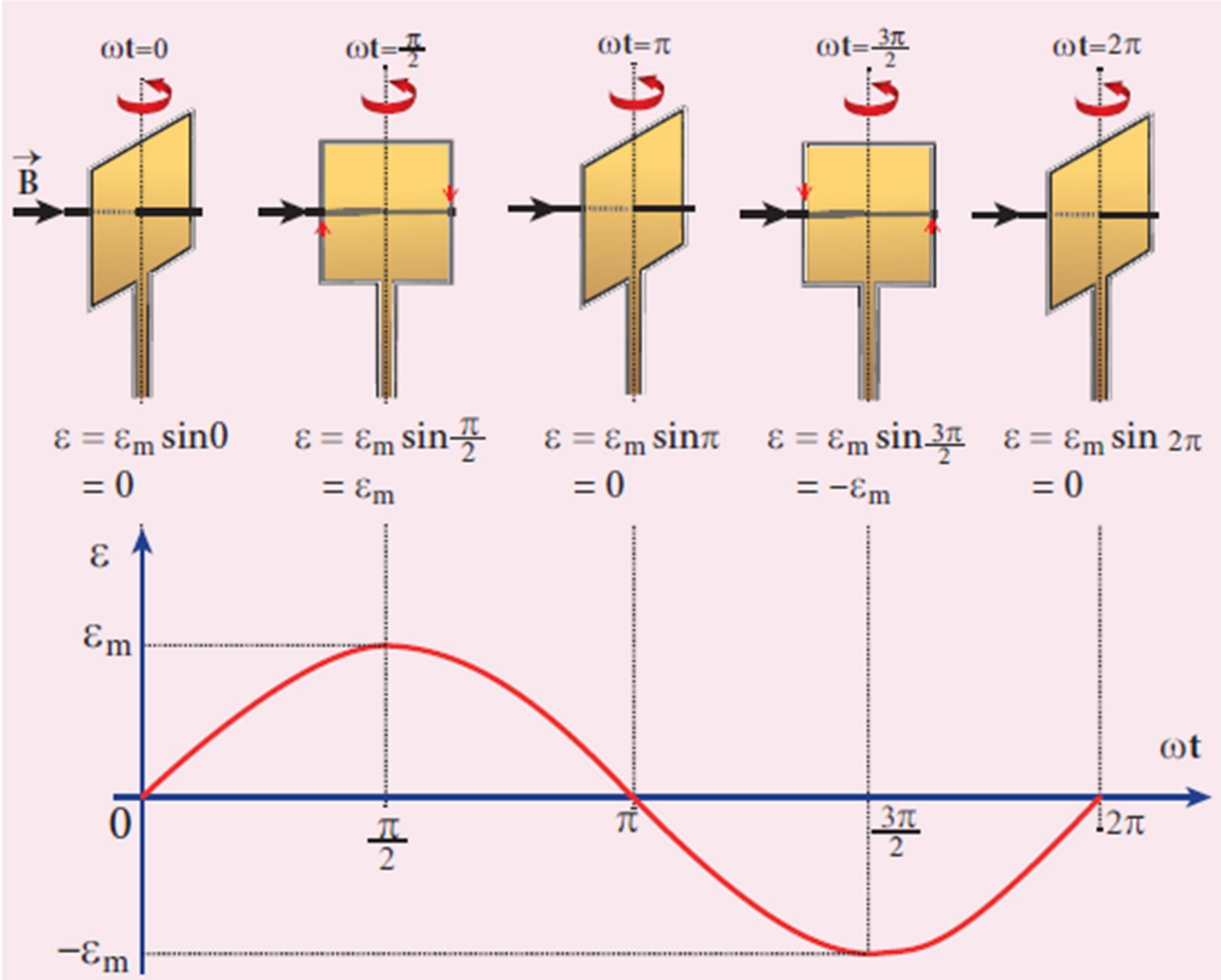


Figure 4.26 Variation of induced emf as a function of ωt

4.5 AC generator

4.5.1 Introduction

- AC generator or alternator is an energy conversion device.
- It converts mechanical energy used to rotate the coil or field magnet into electrical energy.

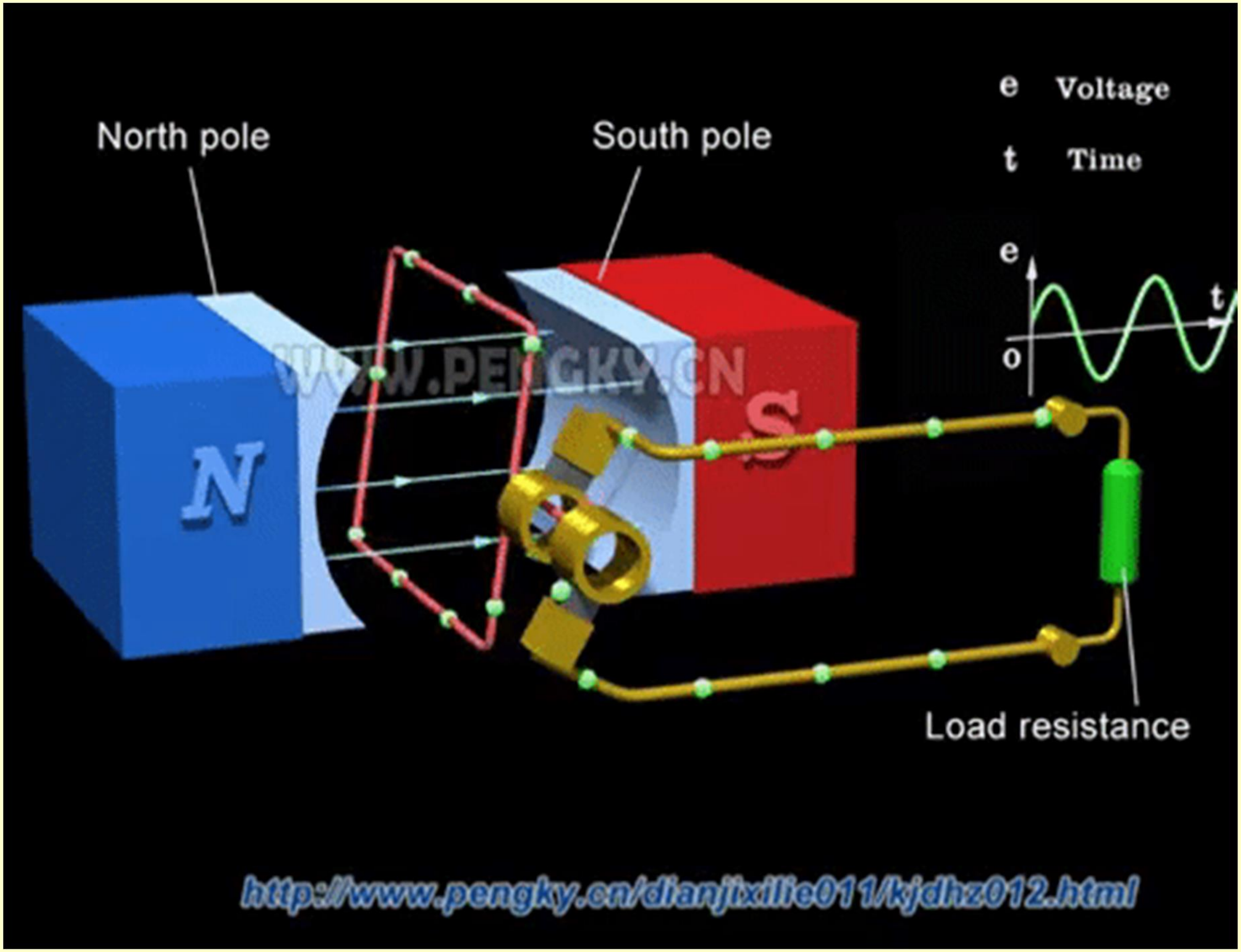
4.5.2 Principle

- Principle - Electromagnetic induction
- Magnitude of the induced emf - Faraday’s law
- Its direction - Fleming’s right hand rule

Note

Alternating emf is generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil.

The first method is used for small AC generators while the second method is employed for large AC generators. The rotating-field method is the one which is mostly used in power stations.


e Voltage
t Time
North pole
South pole
WWW.PENGKY.CN
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Load resistance
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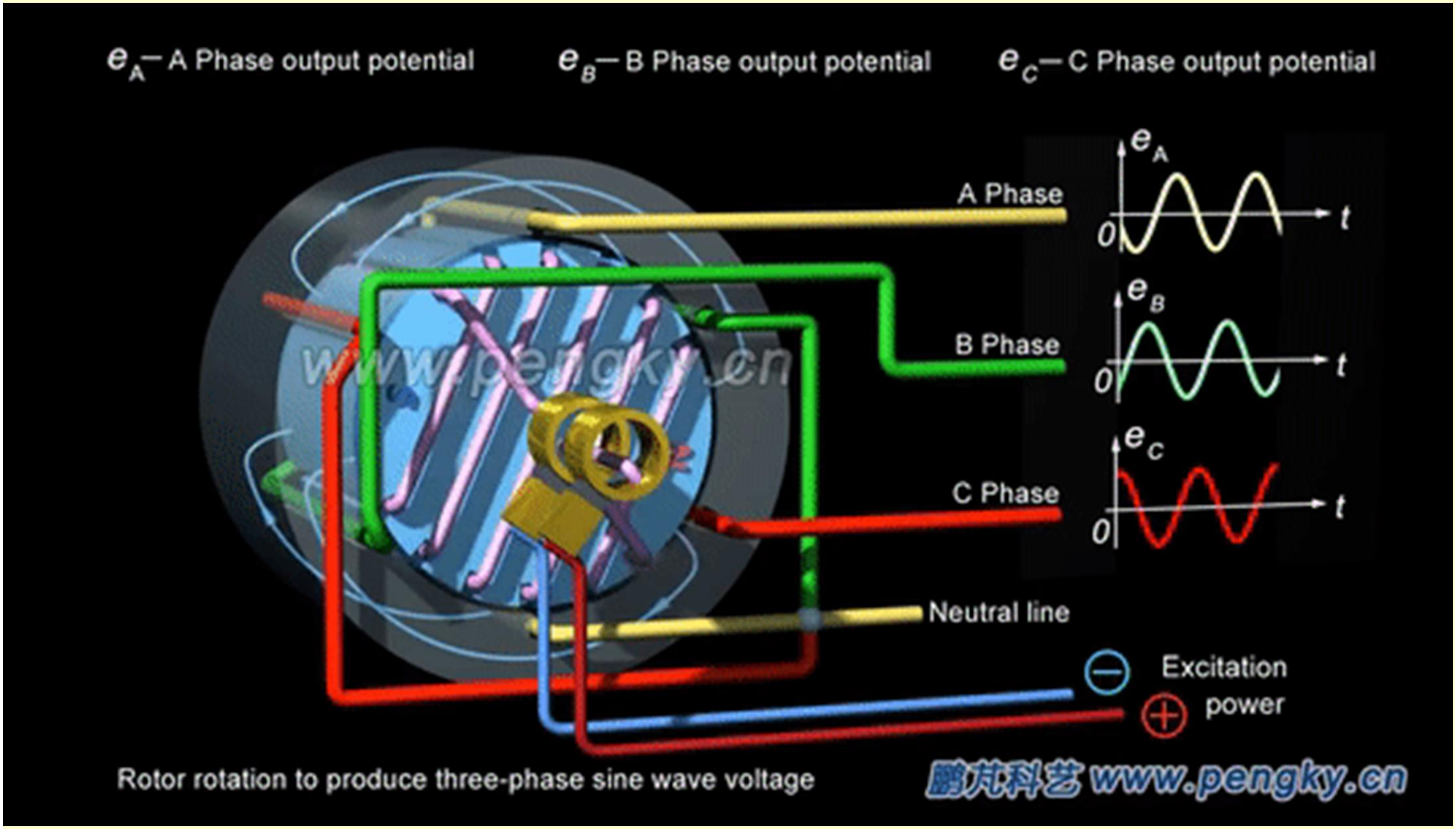
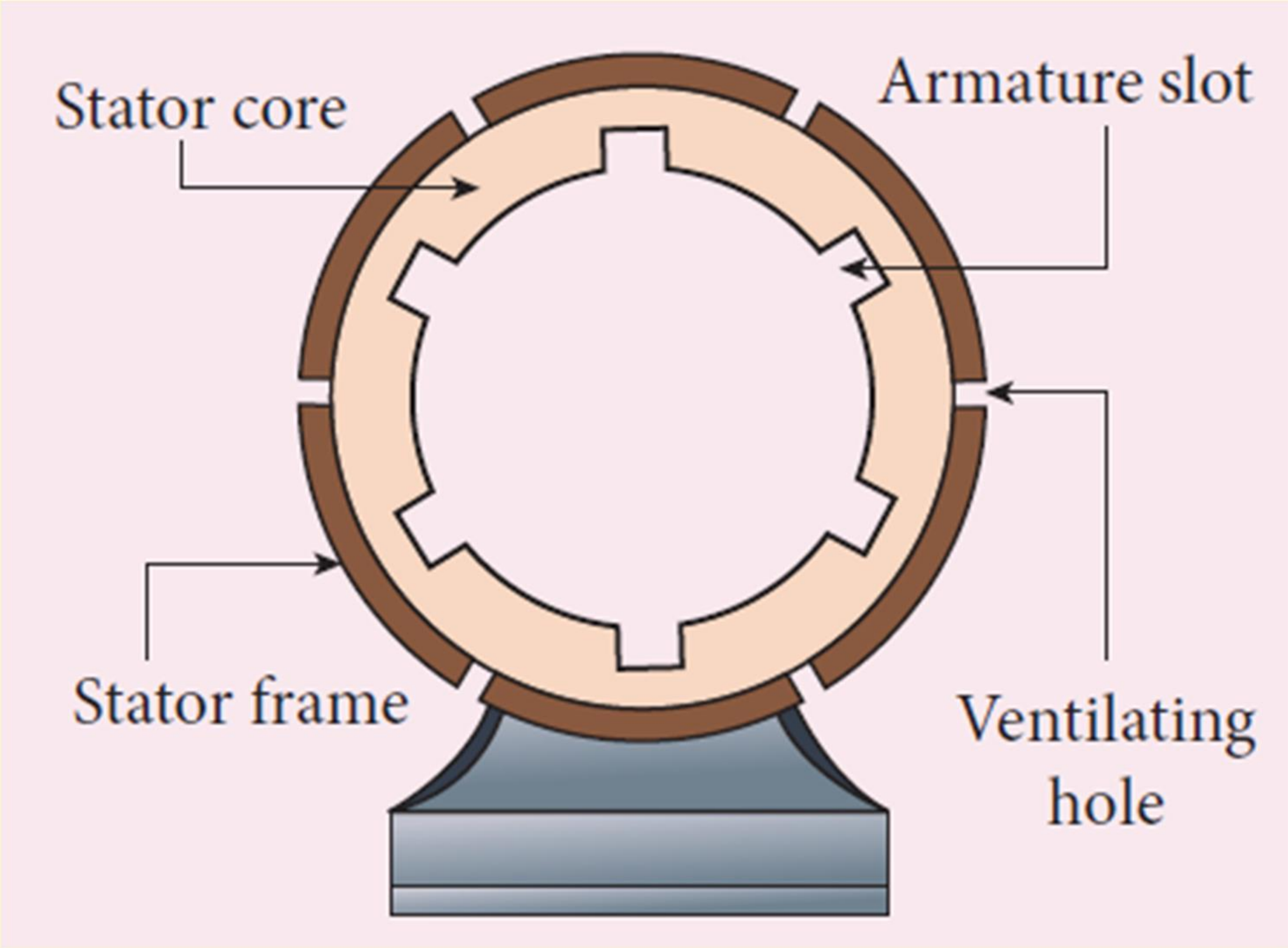


e_A — A Phase output potential
e_B — B Phase output potential
e_C — C Phase output potential
A Phase
B Phase
C Phase
Neutral line
Excitation power
e_A
e_B
e_C
0
t
www.pengky.cn
Rotor rotation to produce three-phase sine wave voltage
鹏芃科艺 www.pengky.cn

4.5.3 Construction

- Two major parts:
  - Stator – stationary
  - Rotor – rotates inside the stator
- In any standard construction of commercial alternators,
  - the armature winding – stator
  - the field magnet – rotor

- Stator:
  - Stationary part
  - It has three components namely, Stator frame, Stator core and Armature winding

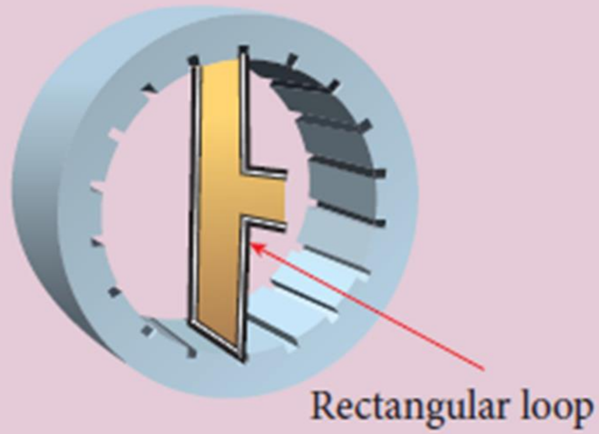


Construction of AC generator (Not for examination)

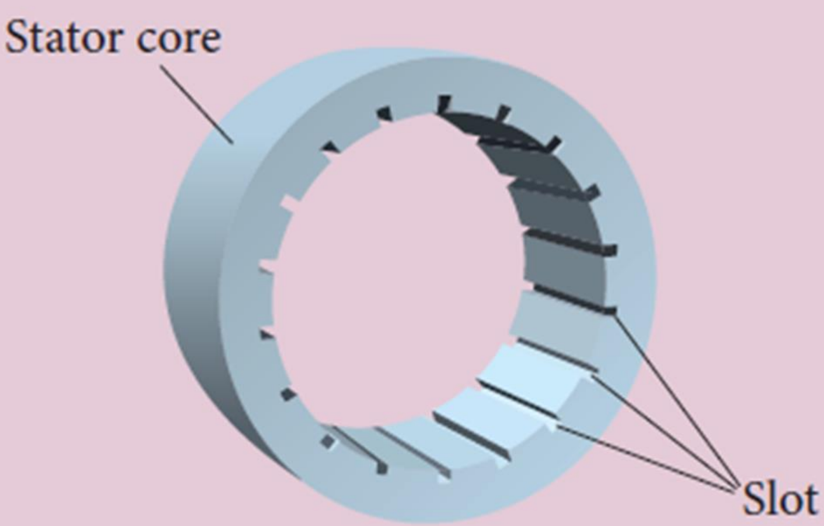
Alternator consists of two major parts, namely stator and rotor. (This box is given for better understanding of constructional details)

i) Stator

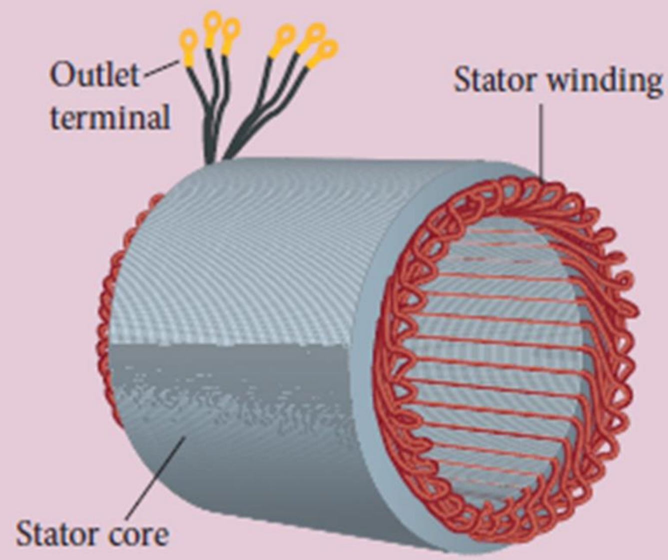
Stator has three components, namely stator frame, stator core and armature winding.



Figure(b): Stator core with rectangular loop



Figure(a): Stator core with empty slots



Figure(c): Stator core with armature windings

Stator winding of a generator at a hydroelectric power station

- Rotor:
  - It rotates inside the stator
  - It contains magnetic field windings
  - DC source – To magnetize the field windings
  - Slip rings – Two ends of the winding are connected
  - Two brushes - To maintain connection between DC source and field windings
  - 2 types of rotors
    - Salient pole rotor and
    - Cylindrical pole rotor.

- Salient pole rotor:

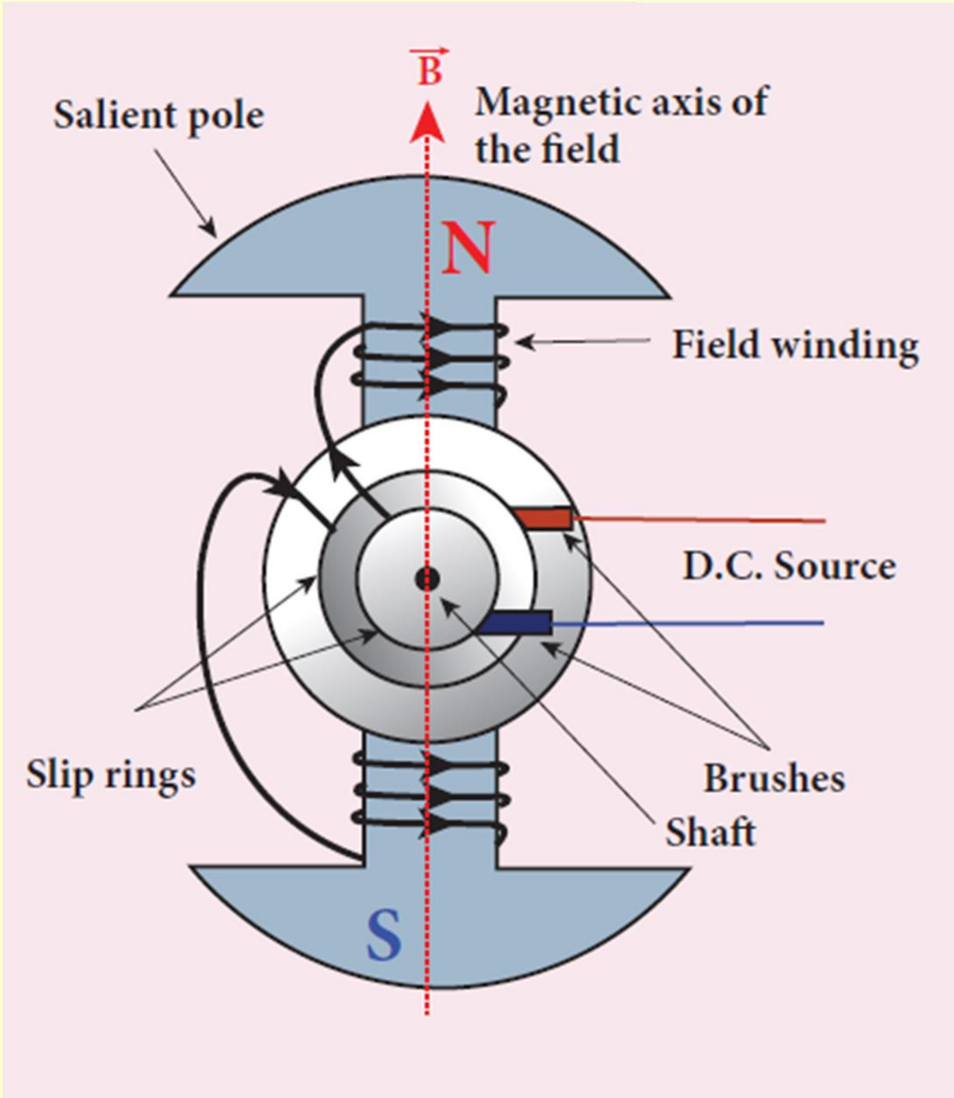


Figure 4.30 Salient 2-pole rotor

- Cylindrical pole rotor:

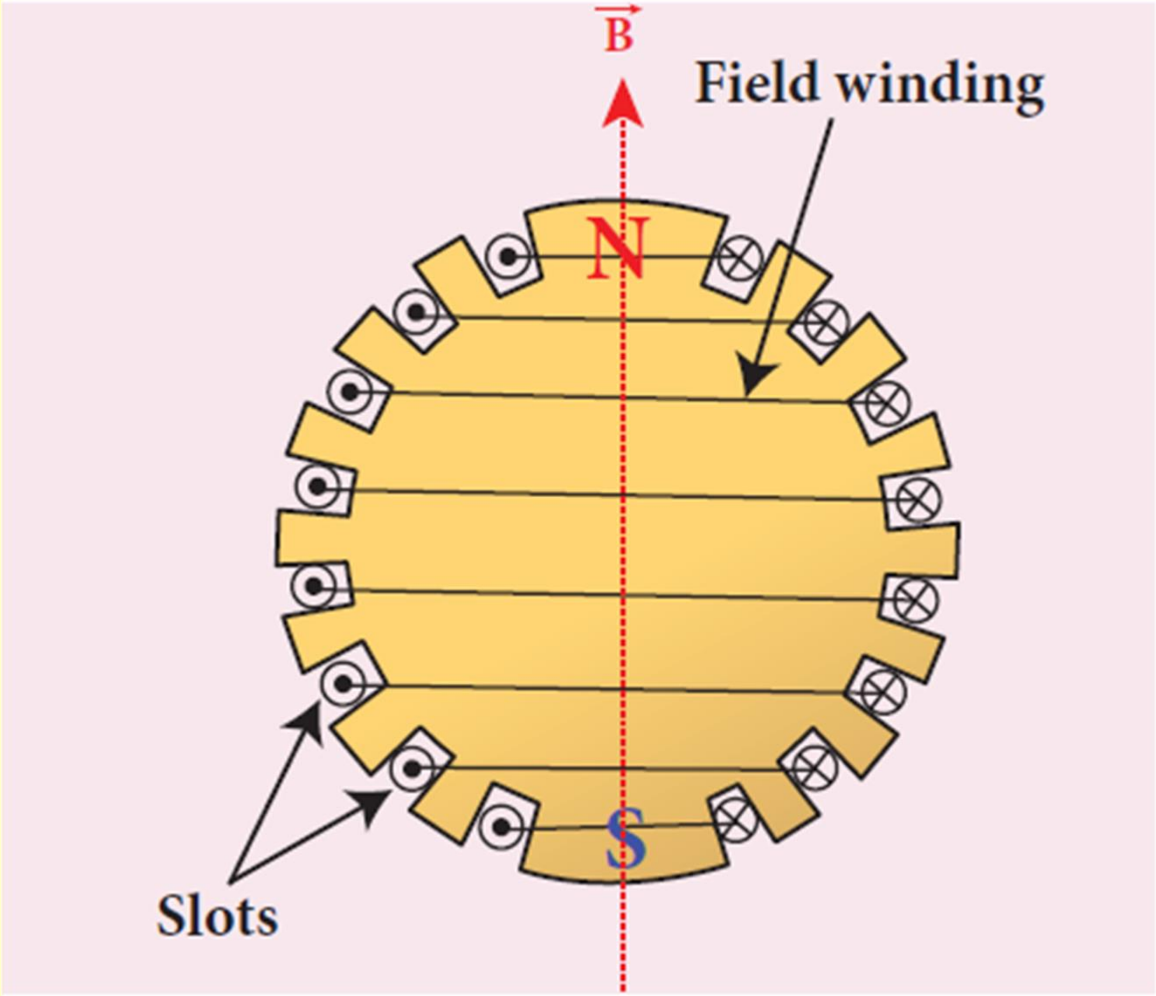
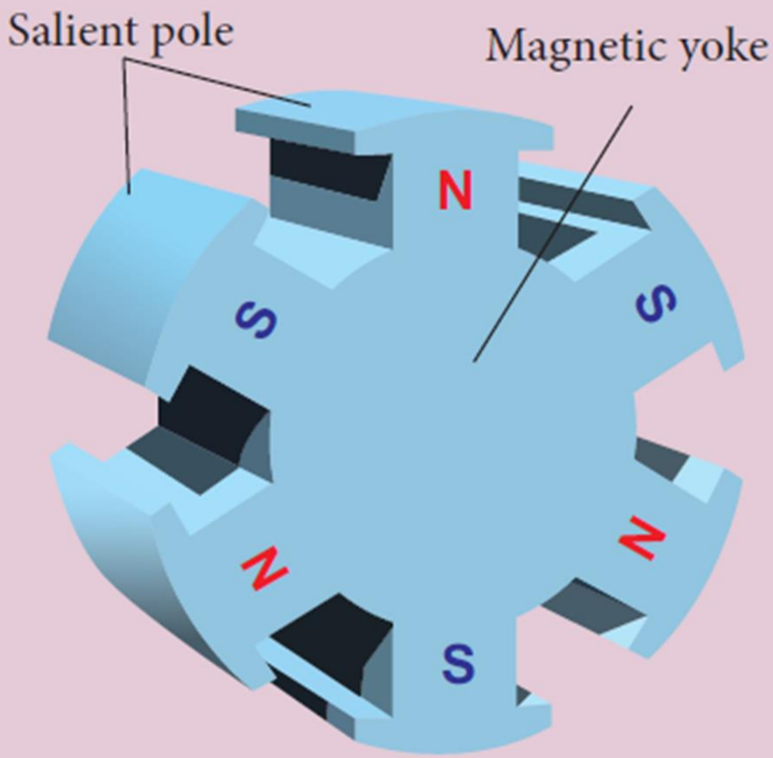


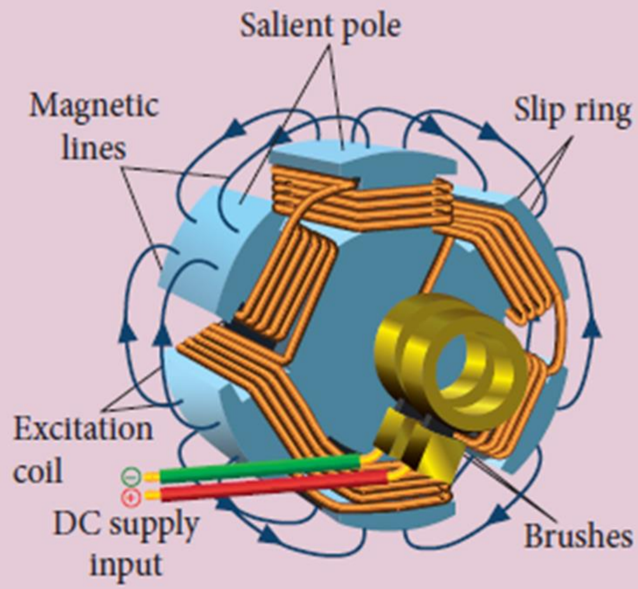
Figure 4.31 Cross-sectional view of cylindrical 2-pole rotor

ii) Rotor

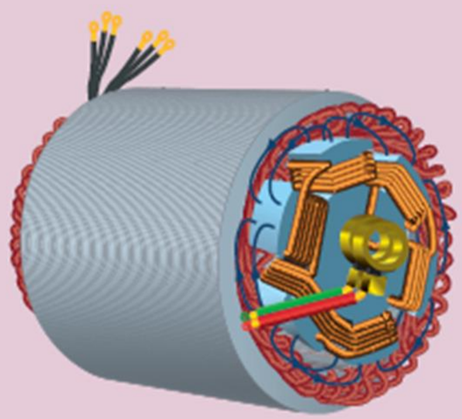
Rotor contains magnetic field windings, slip rings and brushes mounted on the same shaft.



Figure(d): Salient 6-pole rotor



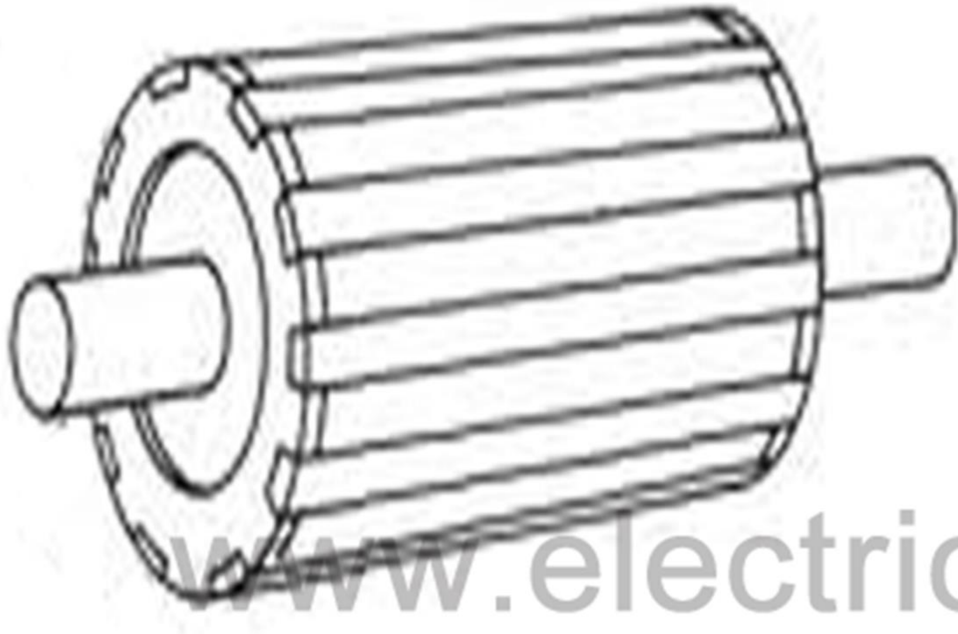
Figure(e): Salient 6-pole rotor with field windings, slip rings and brushes



Figure(f): Stator core and rotor

Salient Pole Type

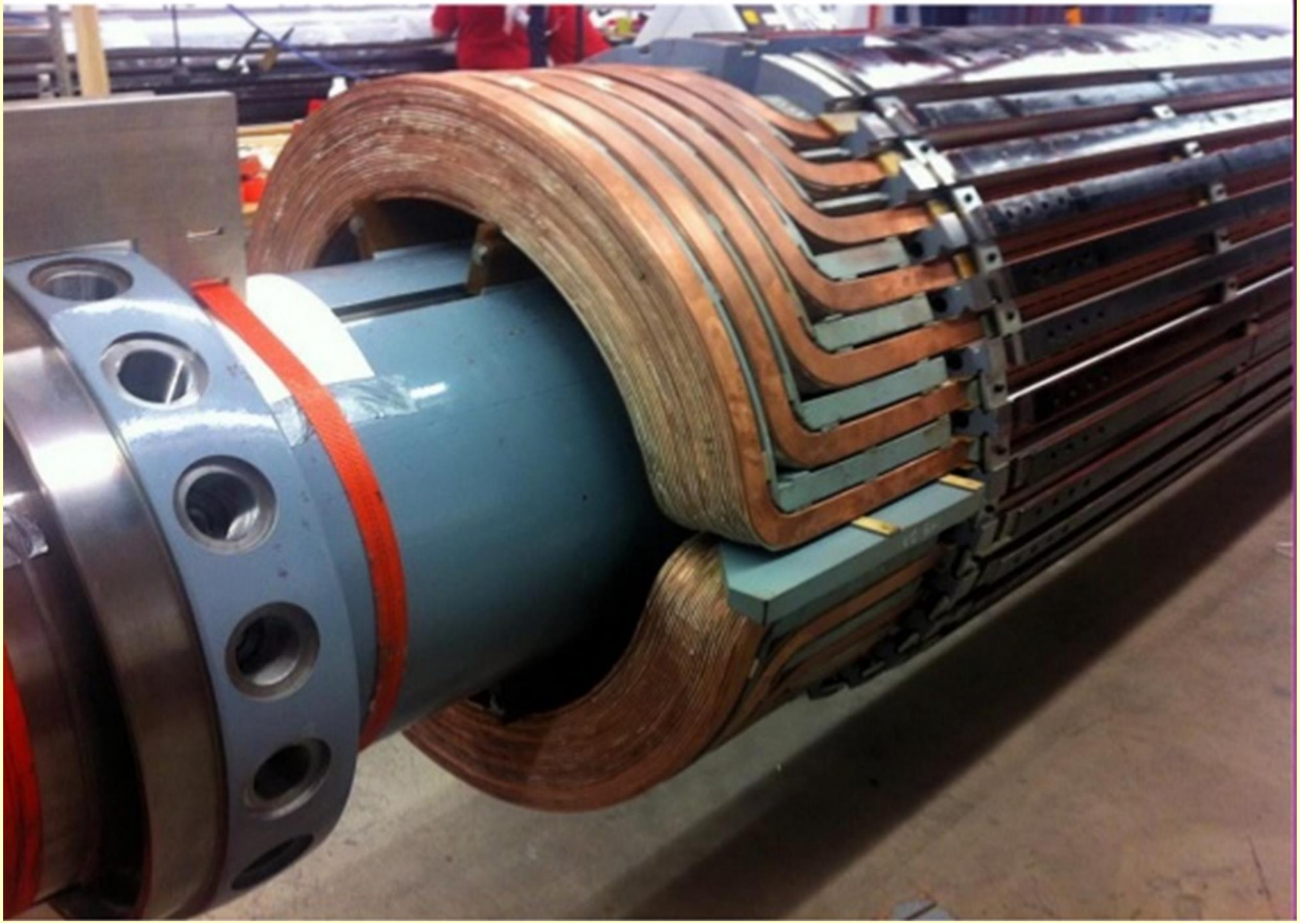
Rotor of a generator at a hydroelectric power station

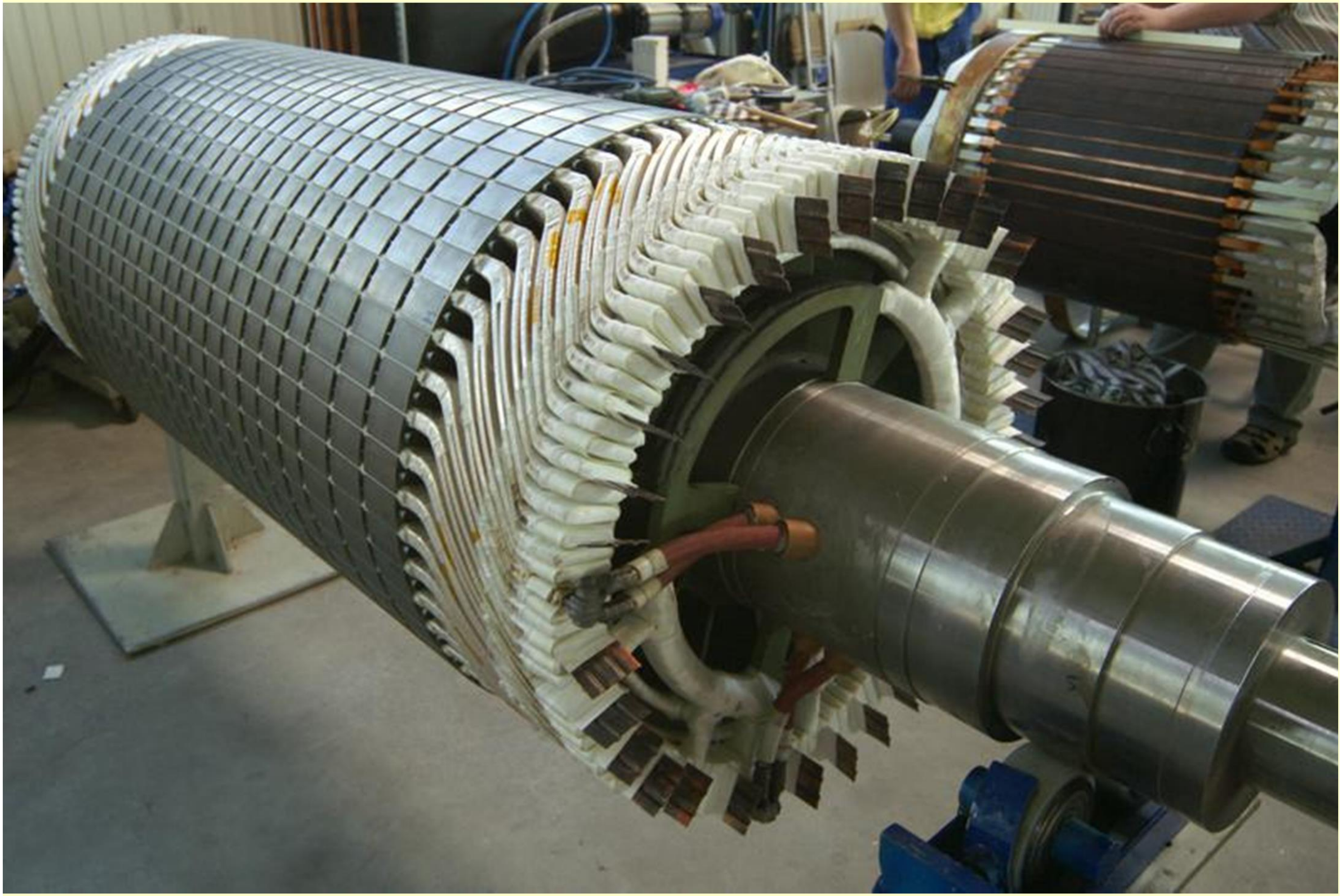


Cylindrical rotor

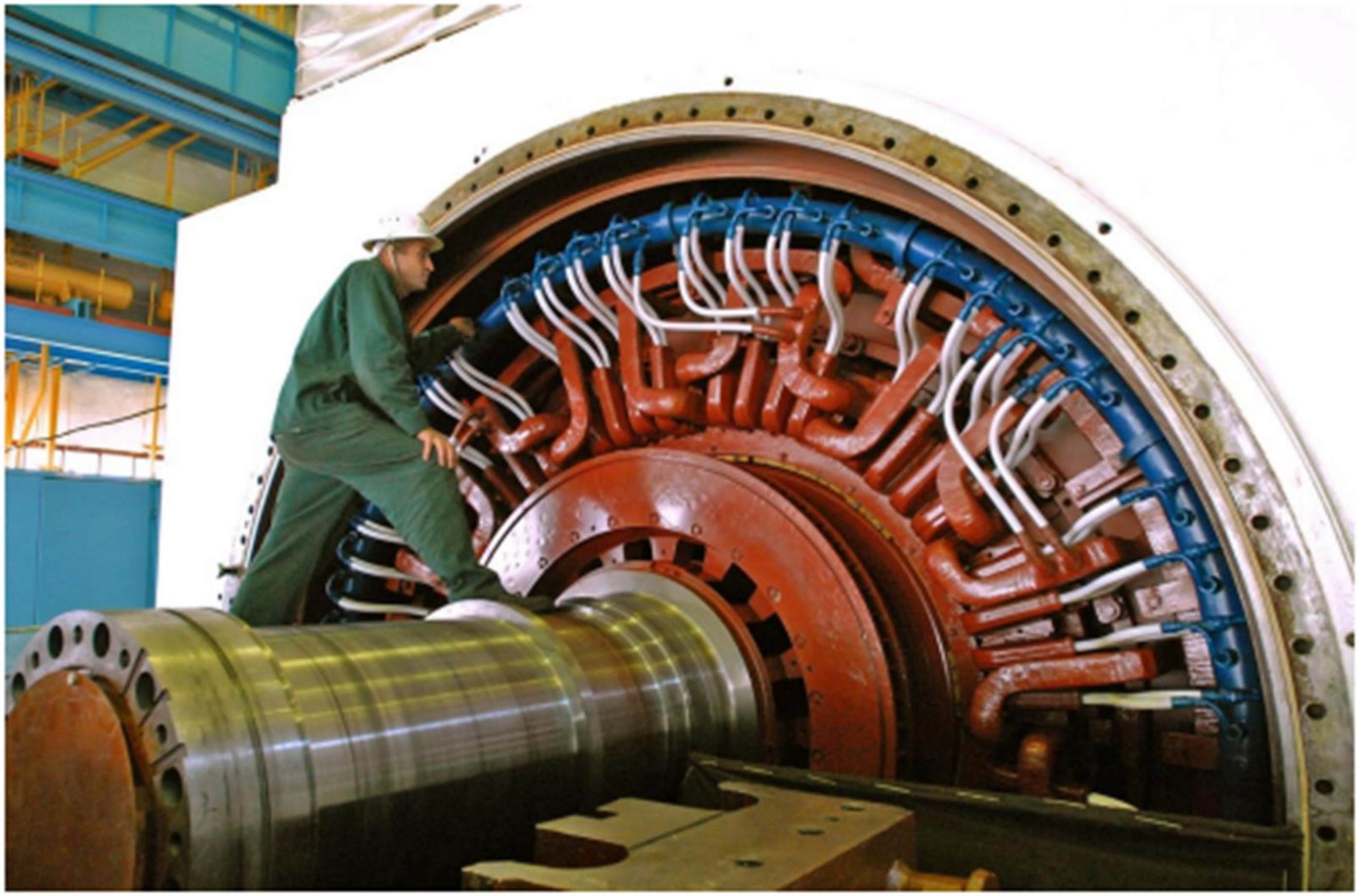
Cross sectional view

CYLINDRICAL ROTOR





Cylindrical Rotor Type



4.5.4 Advantages of stationary armature-rotating field alternator

- The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
- The insulation of stationary armature winding is easier.
- The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
- Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

4.5.5 Single phase AC generator

- Armature conductors are connected in series so as to form a single circuit
- This generates a single-phase alternating emf
- Hence it is called single-phase AC generator

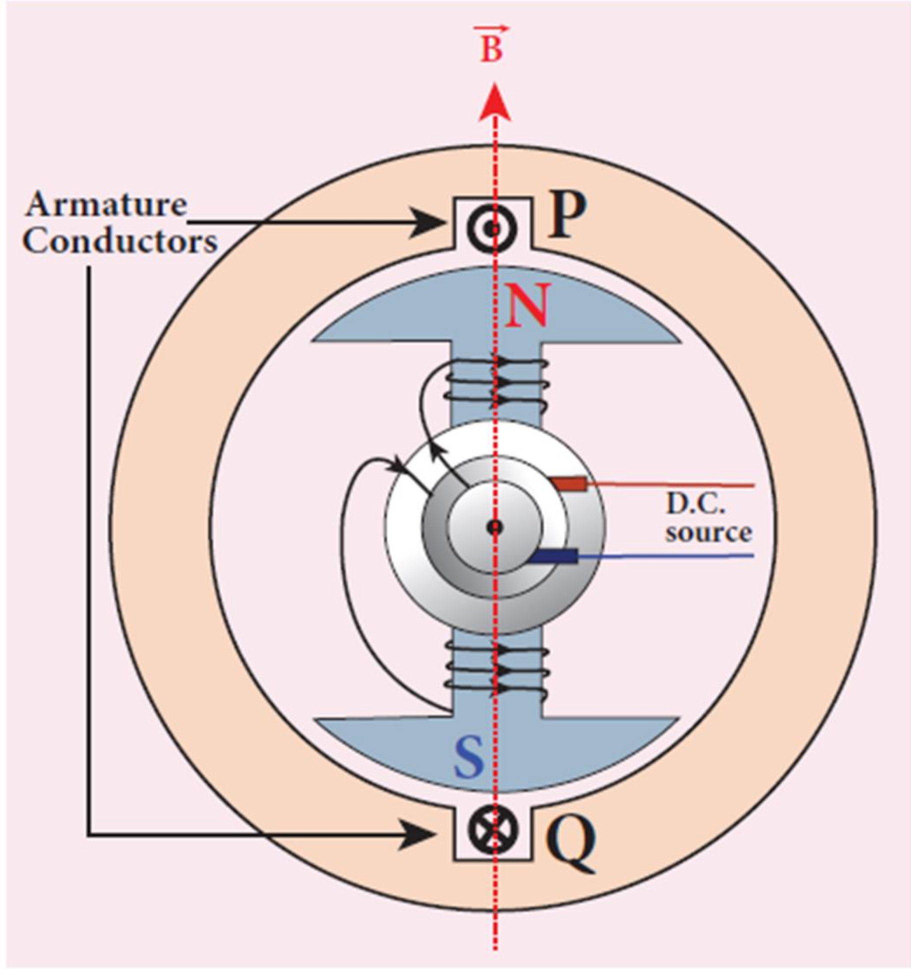


Figure 4.32 Stator core with a rectangular loop and 2-pole rotor

Working:

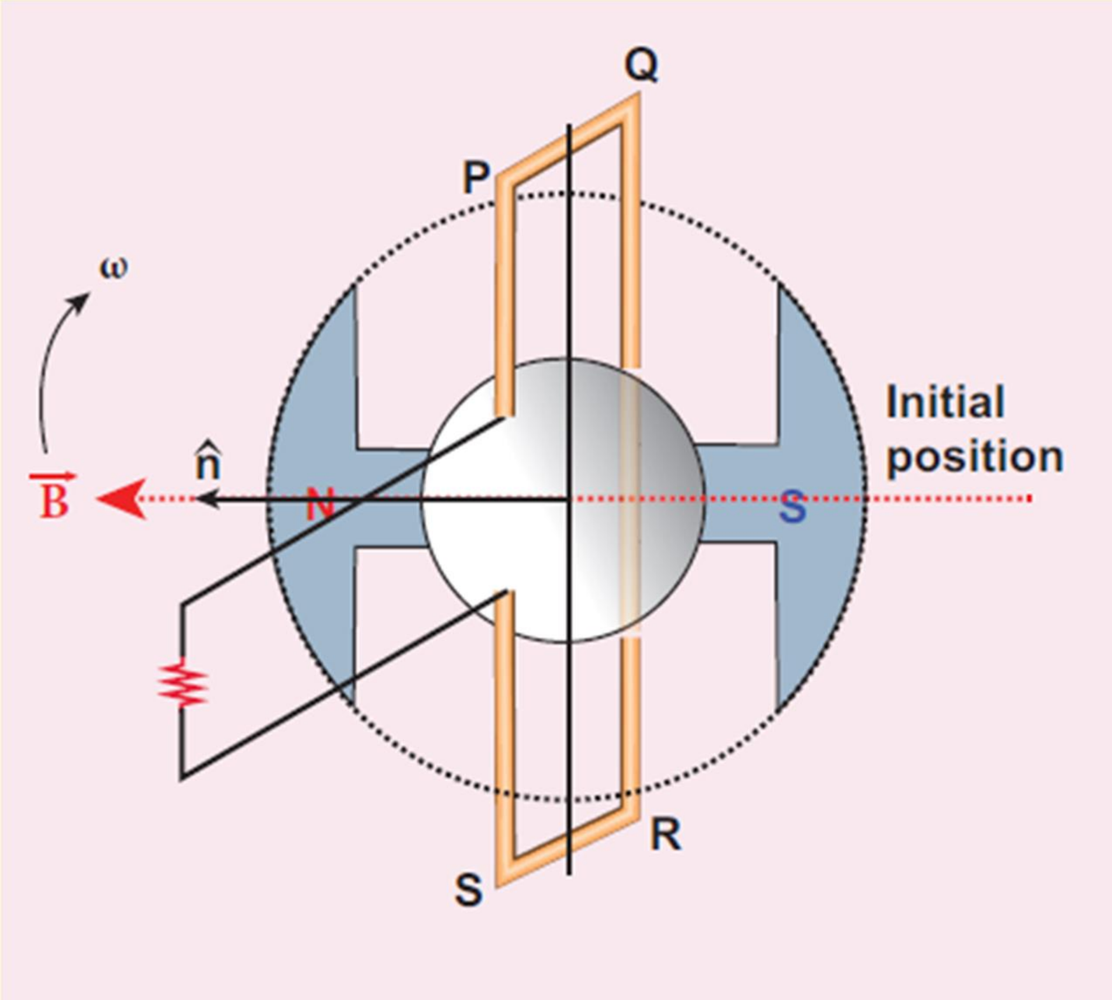


Figure 4.33 The loop PQRS and field magnet in its initial position

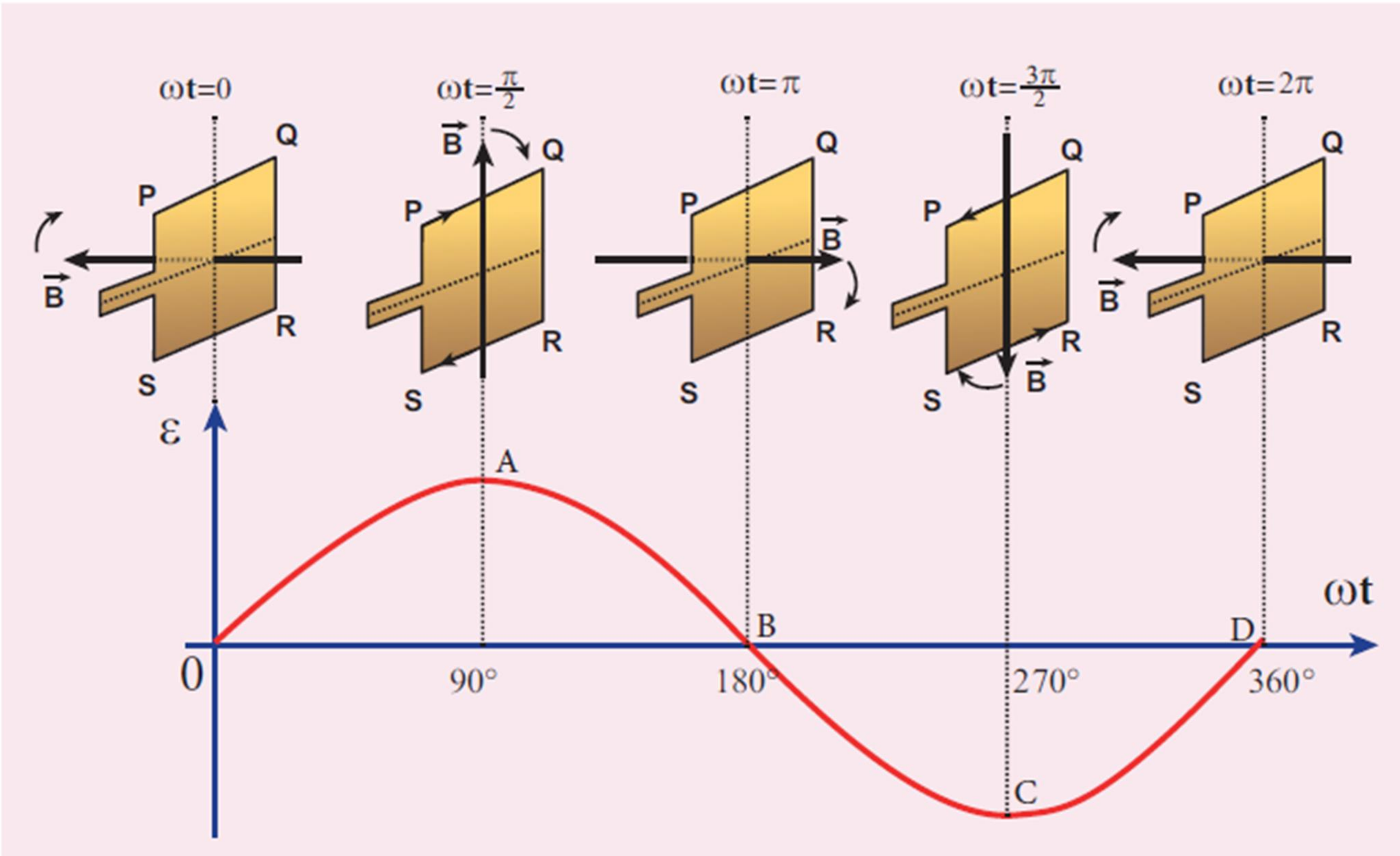
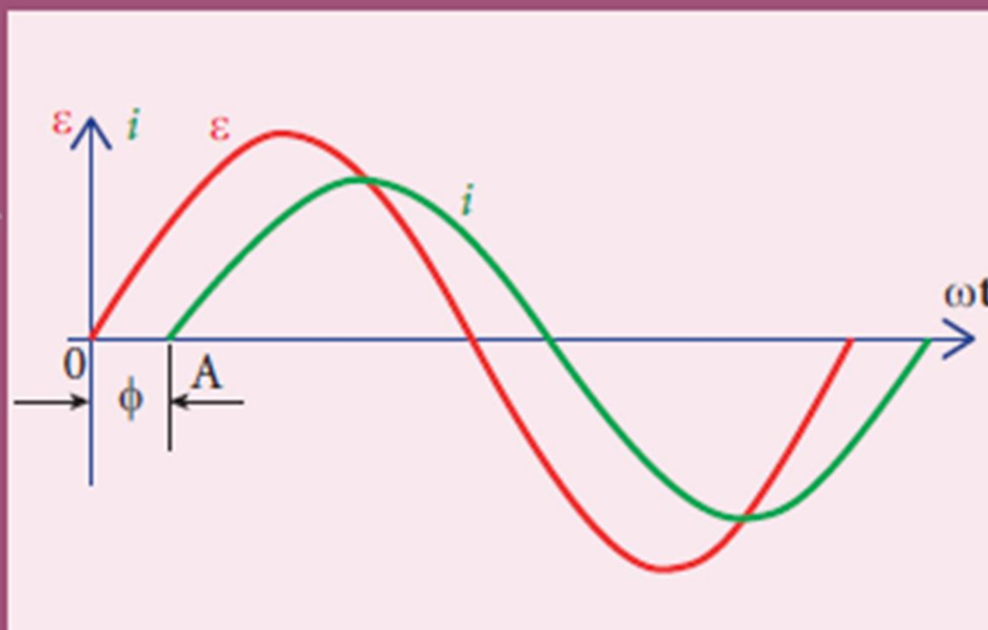


Figure 4.34 Variation of induced emf with respect to time angle

Note

Phase difference

If two alternating quantities of same frequency do not pass through a particular point, say zero point, in the same direction at the same instant, they are said to have a phase difference. The angle between zero points is the angle of phase difference.



4.5.5 Three phase AC generator

- Poly-phase generators – more than one coil
- Two-phase generators – 2 coils and 2 emfs
- Three-phase generators – 3 coils and 3 emfs

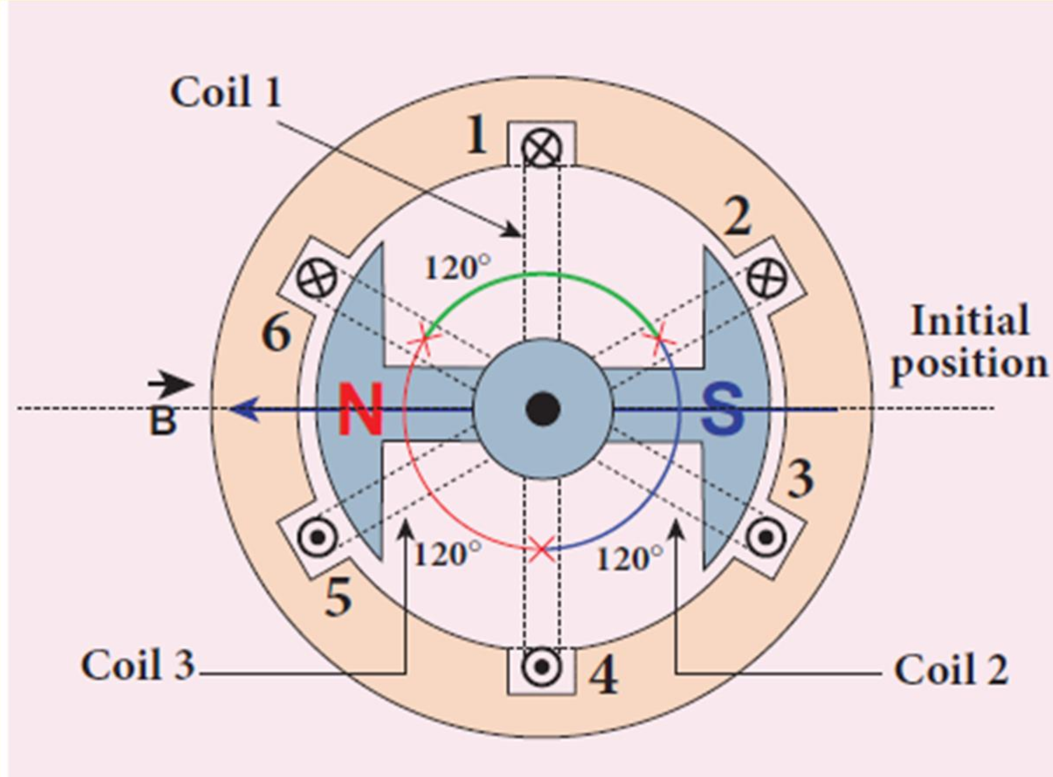


Figure 4.35 Construction of three-phase AC generator

4.5.5 Three phase AC generator

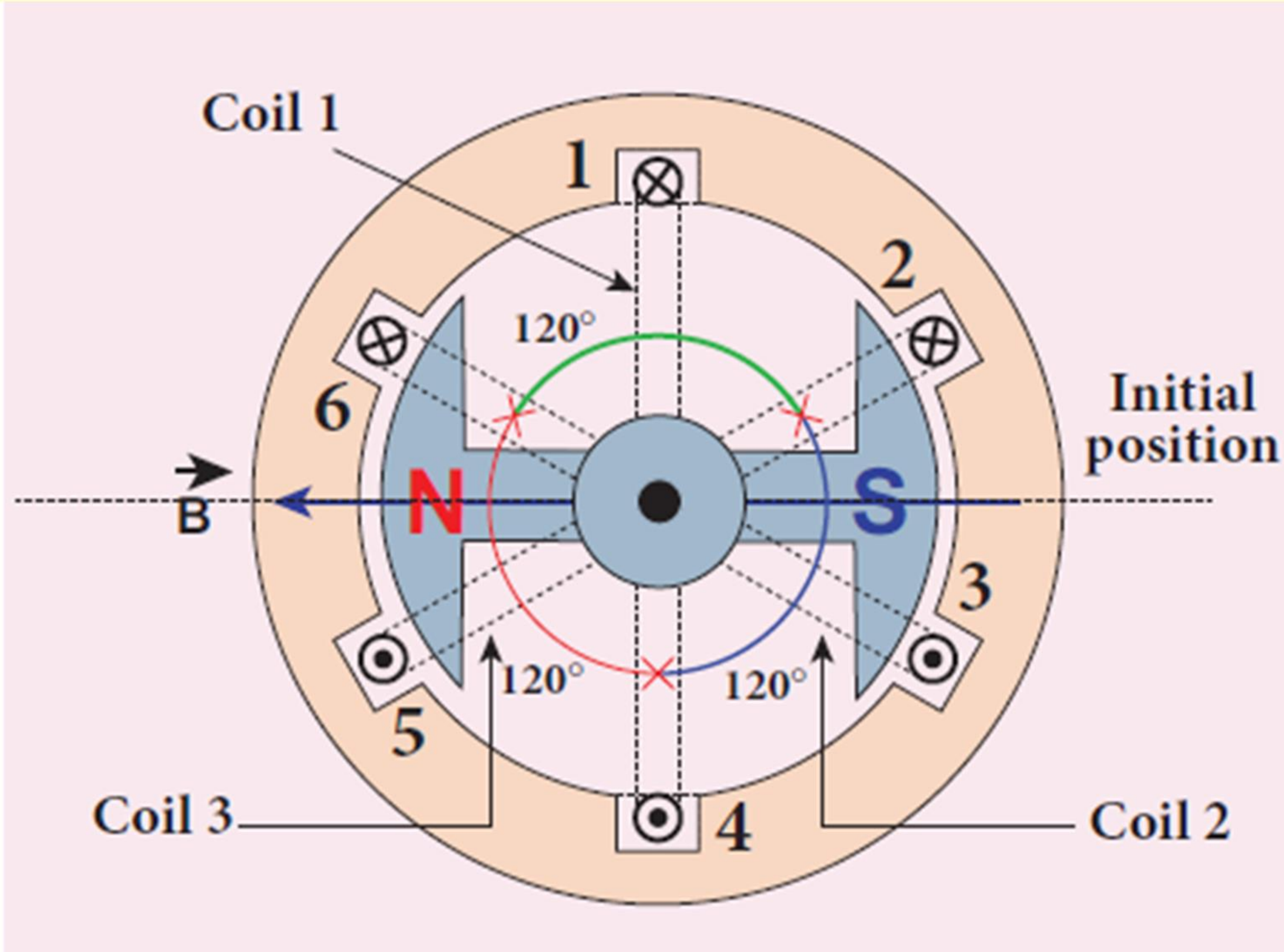


Figure 4.35 Construction of three-phase AC generator

4.5.5 Three phase AC generator

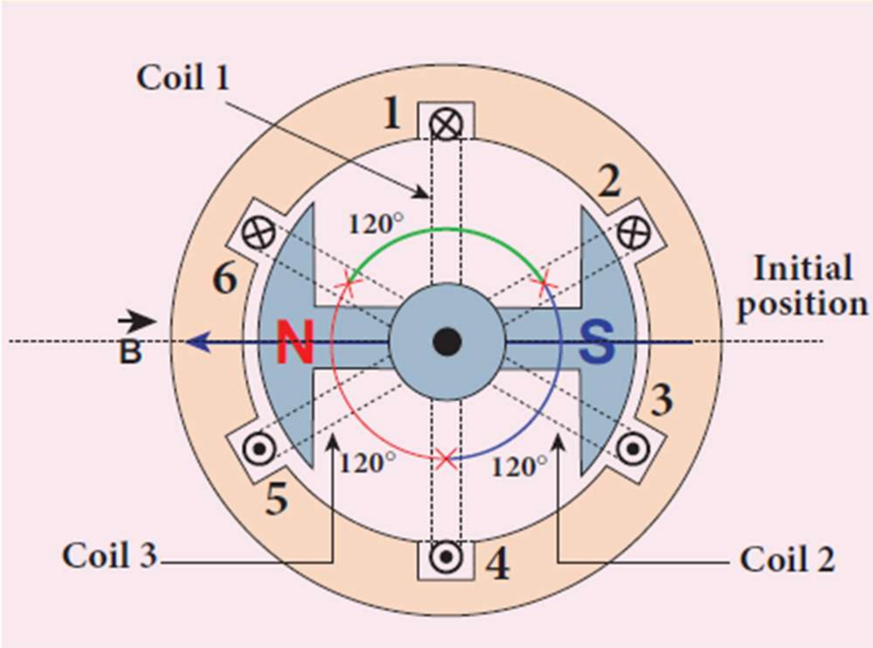


Figure 4.35 Construction of three-phase AC generator

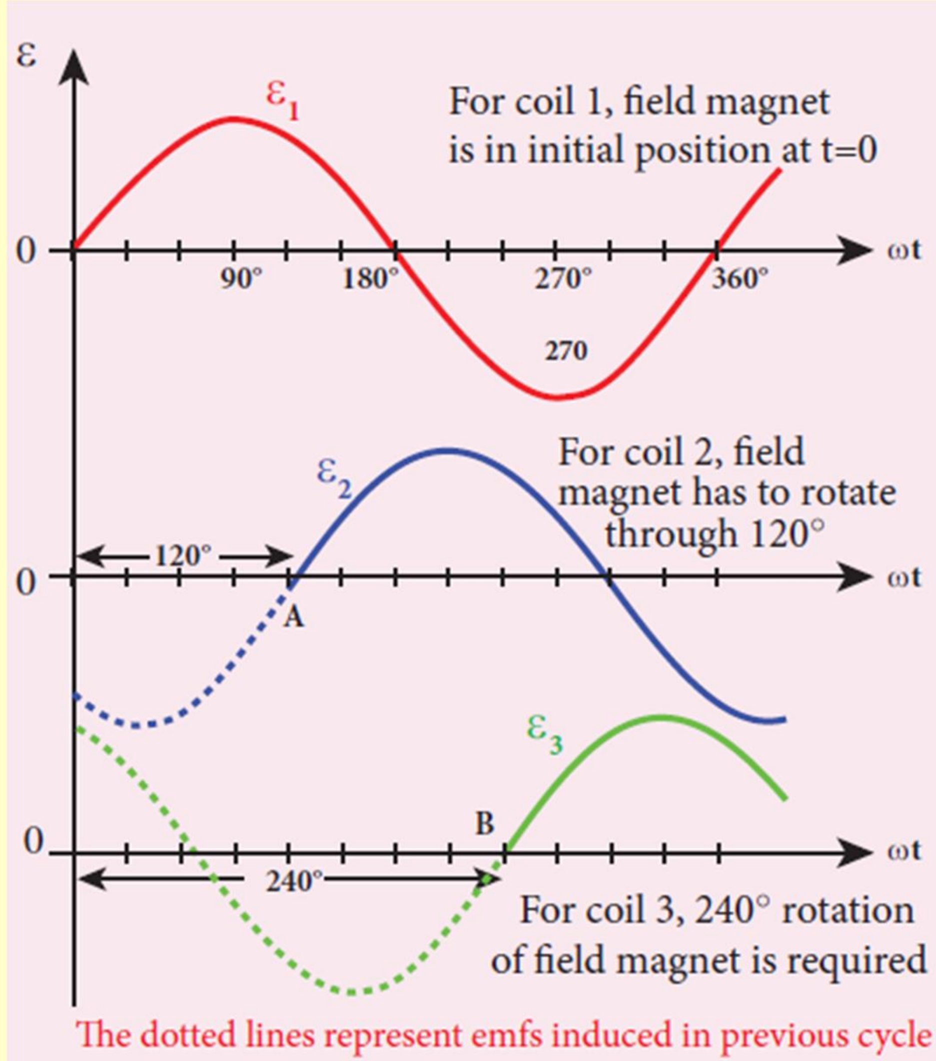
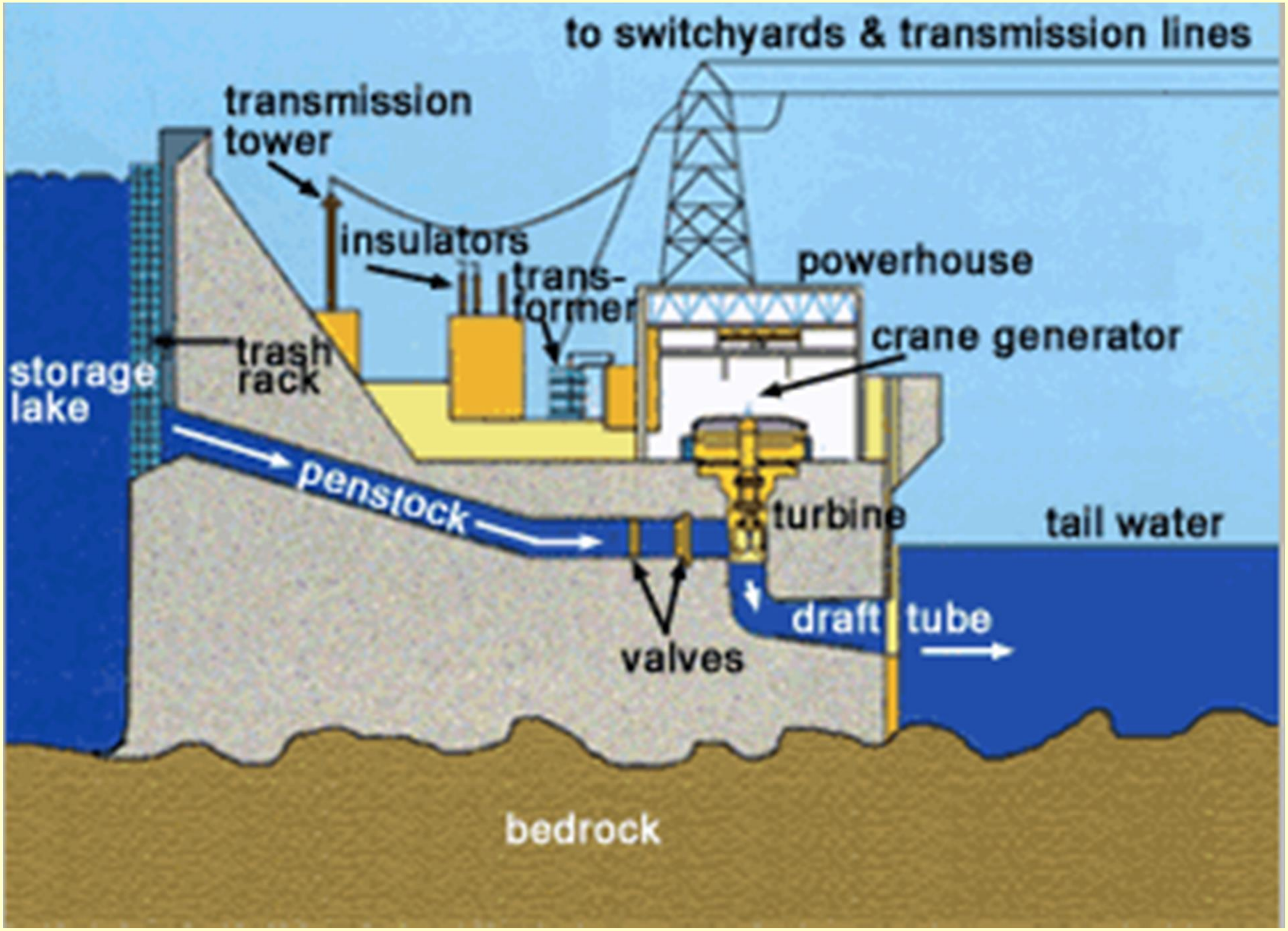
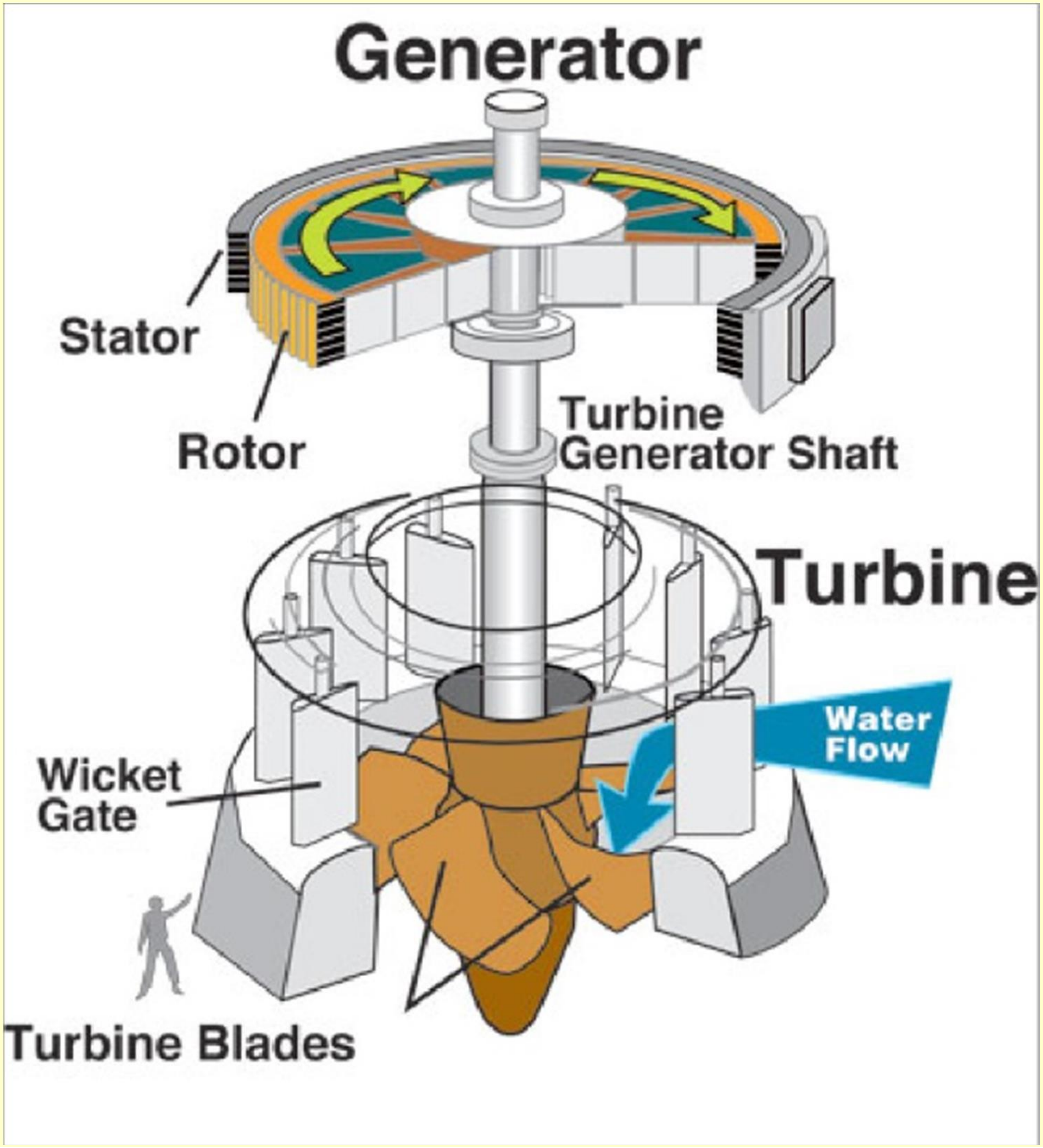


Figure 4.36 Variation of emfs ε_1, ε_2 and ε_3 with time angle.

Cross-sectional view of hydroelectric power station

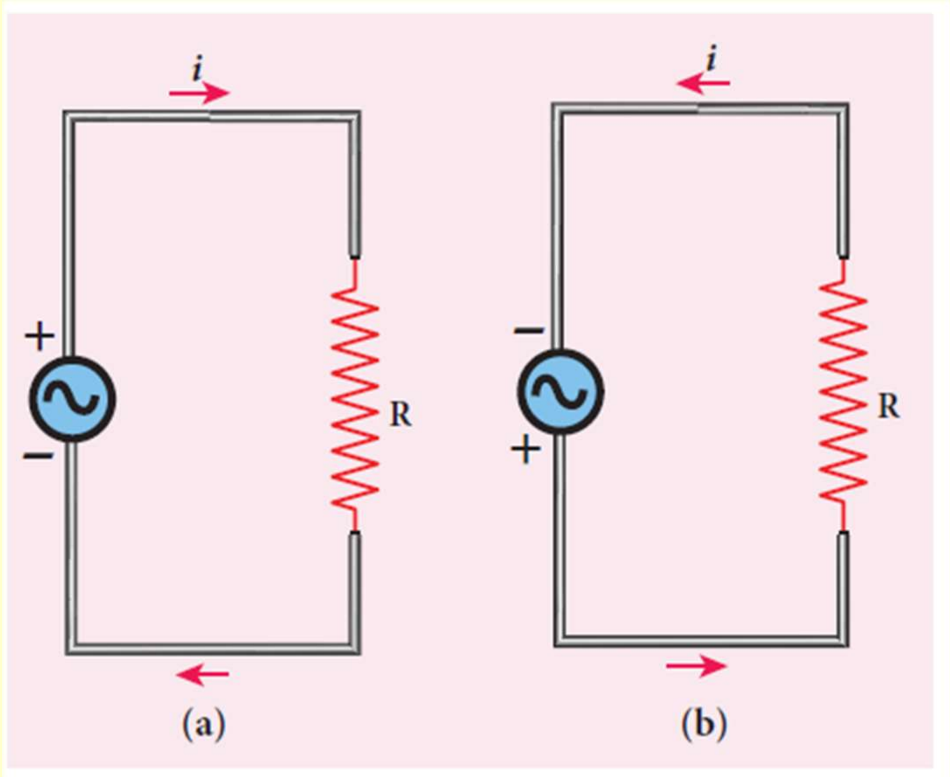



Generator
Stator
Rotor
Turbine
Generator Shaft
Turbine
Water
Flow
Wicket
Gate
Turbine Blades

4.7 Alternating Currents

4.7.1 Introduction

- An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.



Sinusoidal Alternating voltage:

- If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage.

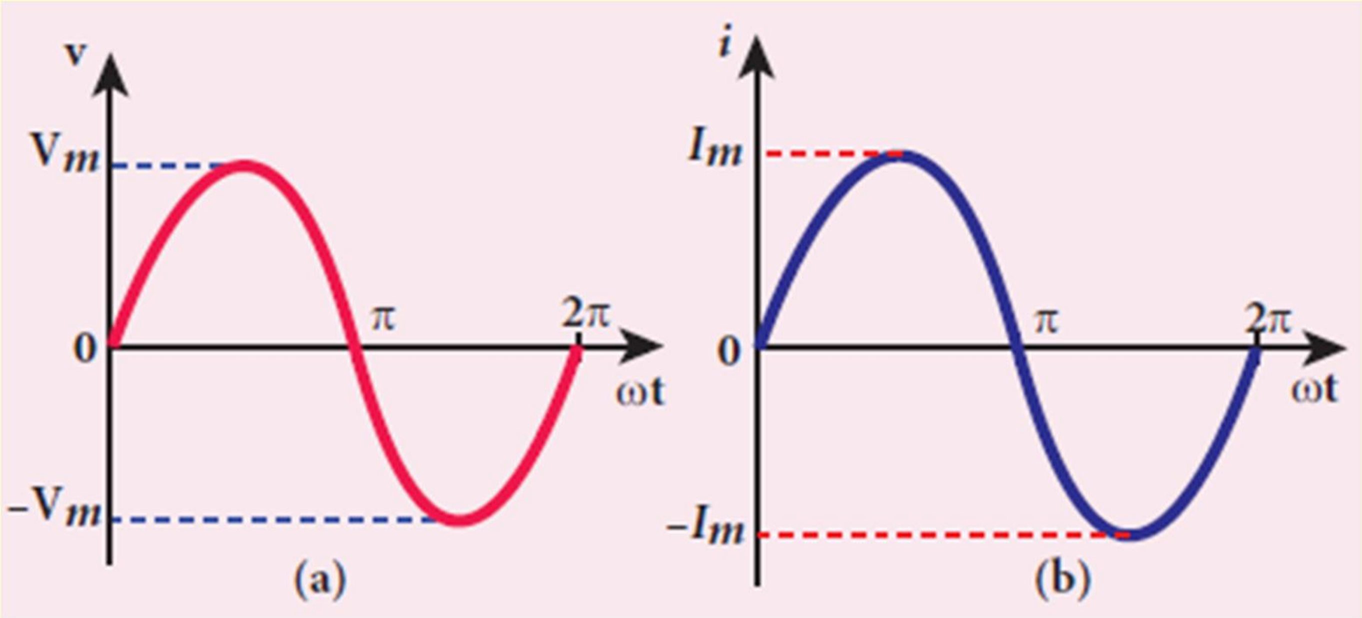
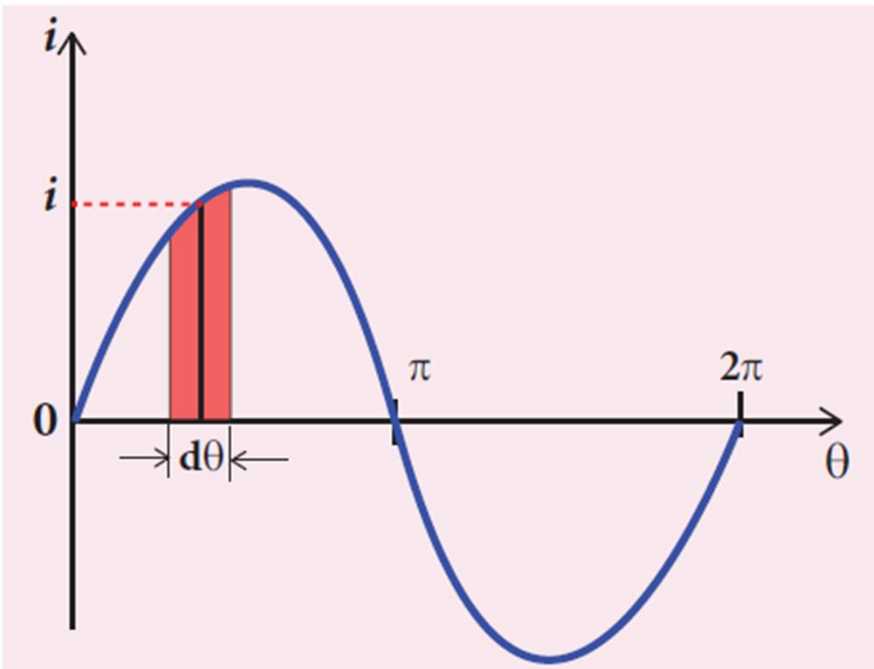


Figure 4.40 (a) Sinusoidal alternating voltage (b) Sinusoidal alternating current

Mean value of AC:

- the average of all values of current over a positive half-cycle or negative half-cycle



$$I_{av} = \frac{\text{Area of positive half-cycle (or negative half-cycle)}}{\text{Base length of half-cycle}} \qquad (4.3$$

Area of the elementary strip = $i\,d\theta$

Area of positive half-cycle

$$= \int_0^{\pi} i d\theta = \int_0^{\pi} I_m \sin\theta d\theta$$

$$= I_m\left[-\cos\theta\right]_0^{\pi} = -I_m\left[\cos\pi - \cos 0\right] \;= 2I_m$$

Substituting this in equation (4.37), we get (The base length of half-cycle is π)

Average value of AC, $I_{av} = \frac{2I_m}{\pi}$

$$I_{av} = 0.637\, I_m \qquad (4.38)$$

Note

For example, if we consider n currents in a half-cycle of AC, namely $i_1, i_2, \ldots i_n$, then average value is given by

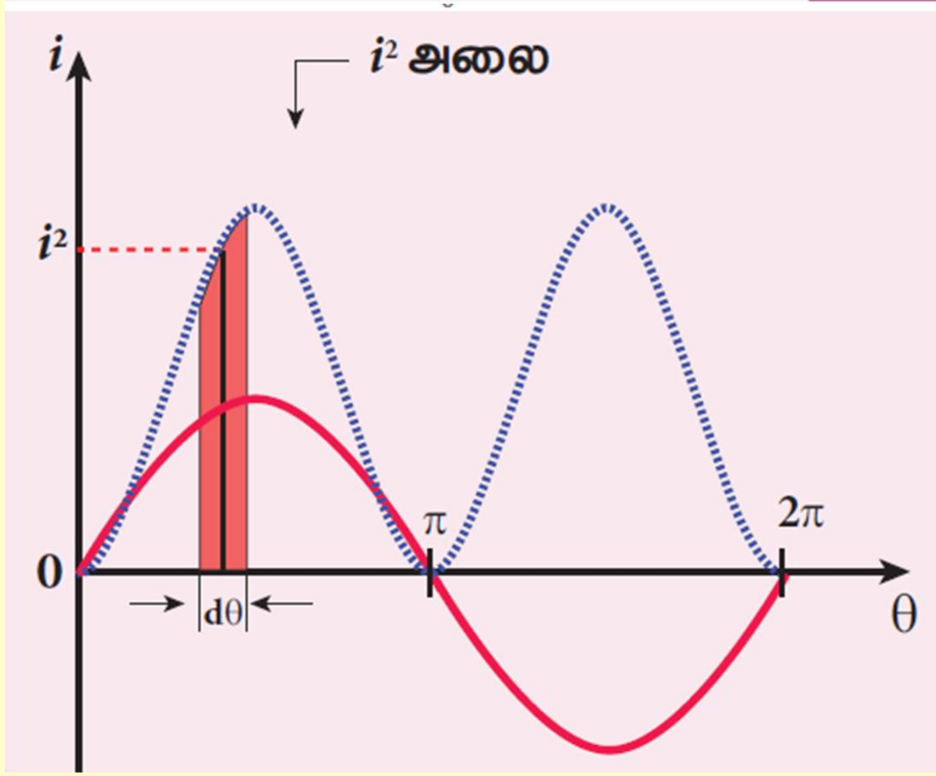
$$I_{av} = \frac{\text{Sum of all currents over half-cycle}}{\text{Number of currents}}$$

$$I_{av} = \frac{i_1 + i_2 + \ldots. + i_n}{n}$$

RMS value of AC:

- the square root of the mean of the squares of all currents over one cycle

$$I_{RMS} = \sqrt{\frac{\text{Area of one cycle of squared wave}}{\text{Base length of one cycle}}} \quad (4.$$



Area of the element = $i^2 d\theta$

Area of one cycle of squared wave $= \int_0^{2\pi} i^2 d\theta$

$$= \int_0^{2\pi} I_m^2 \sin^2\theta d\theta = I_m^2 \int_0^{2\pi} \sin^2\theta d\theta \qquad (4.40)$$

$$= I_m^2 \int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta$$

since $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$= \frac{I_m^2}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta\right]$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2}\left[\left(2\pi - \frac{\sin 2\times 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{I_m^2}{2}\times 2\pi = I_m^2 \pi \qquad [\because \ \sin 0 = \sin 4\pi = 0]$$

Substituting this in equation (4.39), we get

$$I_{RMS} = \sqrt{\frac{I_m^2 \pi}{2\pi}} = \frac{I_m}{\sqrt{2}}$$

[Base length of one cycle is 2π]

$$I_{rms} = 0.707\, I_m \qquad (4.41)$$

Thus we find that for a symmetrical sinusoidal current rms value of current is 70.7 % of its peak value.

Note

RMS value of alternating current is also called effective value and is represented as I_{eff}. It is used to compare RMS current of AC to an equivalent steady current.

RMS value is also defined as that value of the steady current which when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current when flowing through the same circuit for the same time. The effective value of an alternating voltage is represented by V_{eff}

Note

For example, if we consider n currents in one cycle of AC, namely i_1, i_2, ... i_n, then RMS value is given by

$$I_{RMS} = \sqrt{\frac{\text{Sum of squares of all currents over one cycle}}{\text{Number of currents}}}$$

$$I_{RMS} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$$

Phasor:

- A rotating vector to represent sinusoidal alternating voltage (or current)
- A phasor is drawn in such a way that
  - the length of the line segment = V_m (or I_m)
  - its angular velocity ω = the angular frequency of AC voltage (or current)
  - the projection on any vertical axis gives instantaneous value
  - the angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

Phasor diagram:

- the diagram which shows various phasors and their phase relations

Phasor diagram:

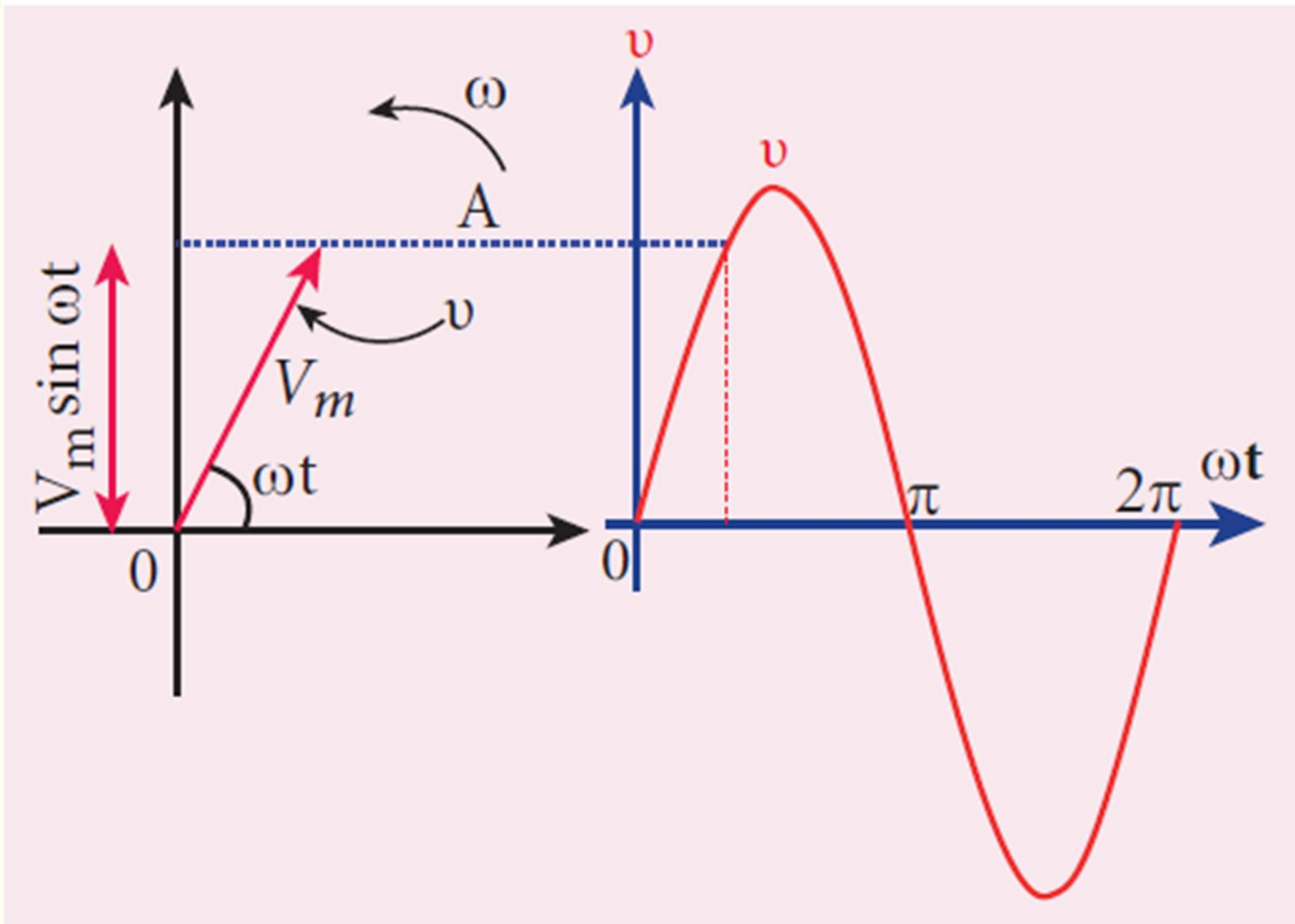
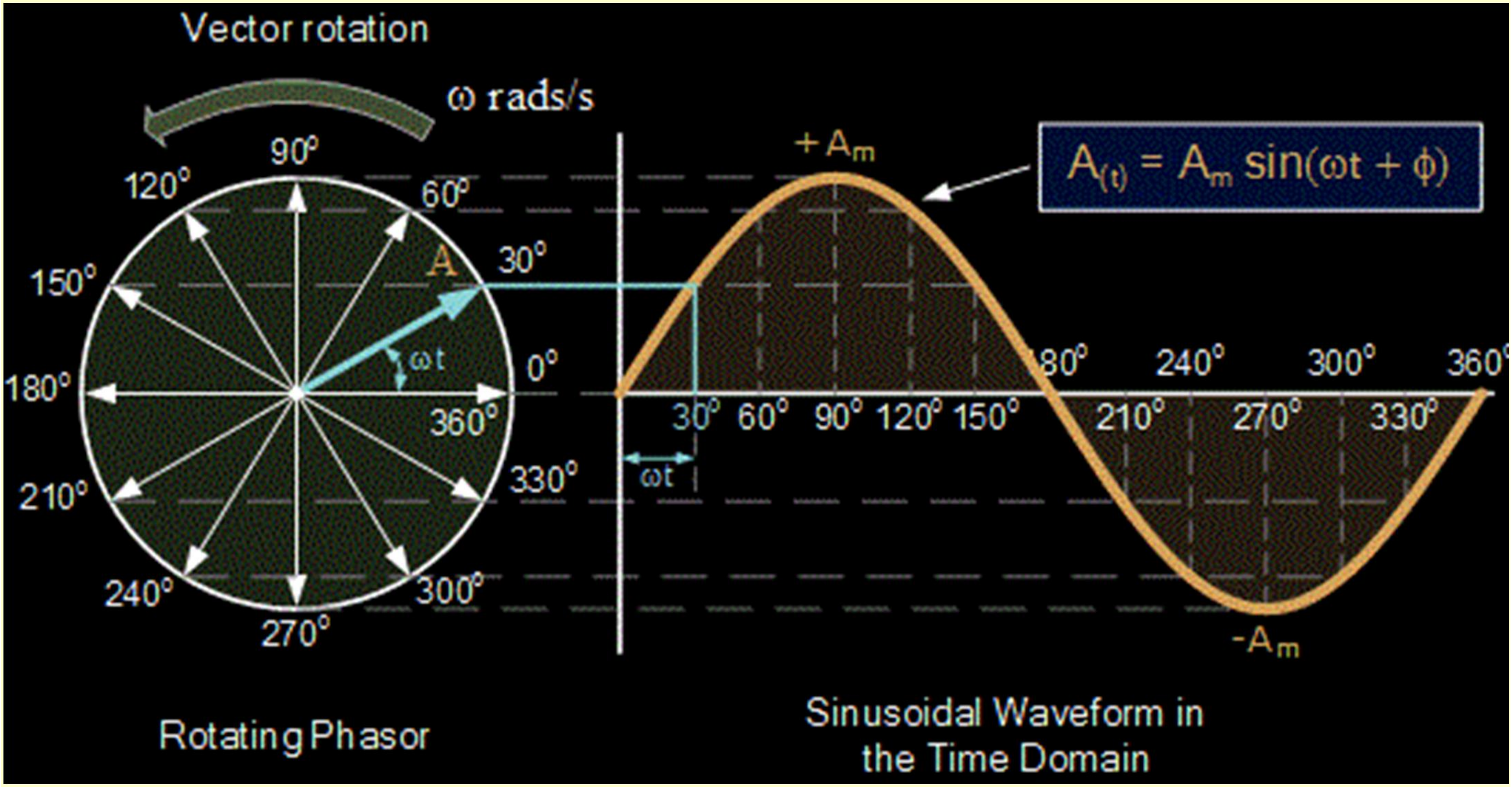
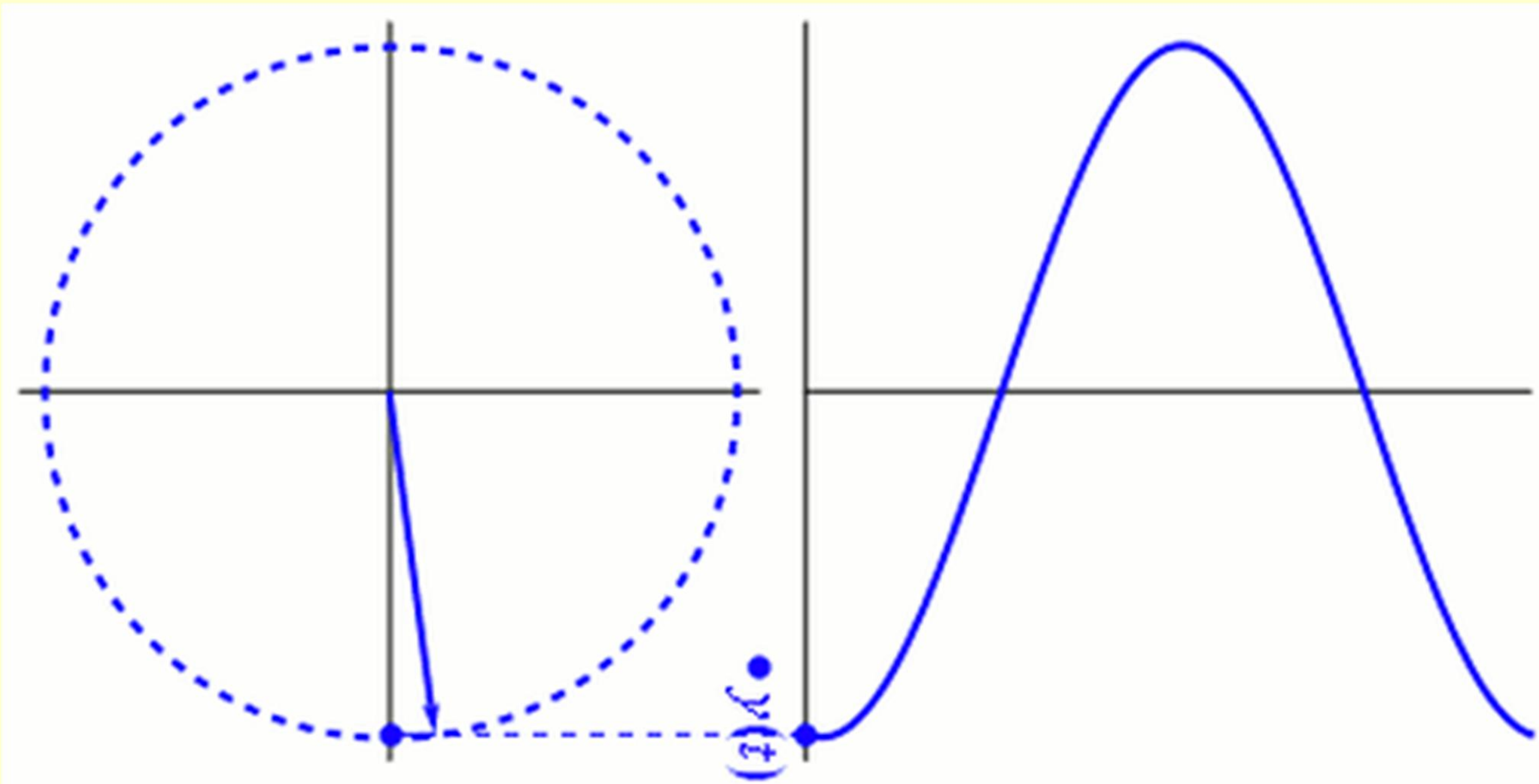


Figure 4.43 Phasor diagram for an alternating voltage $\upsilon = V_m \sin \omega t$

Phasor diagram:



Phasor diagram:



Phasor diagram:

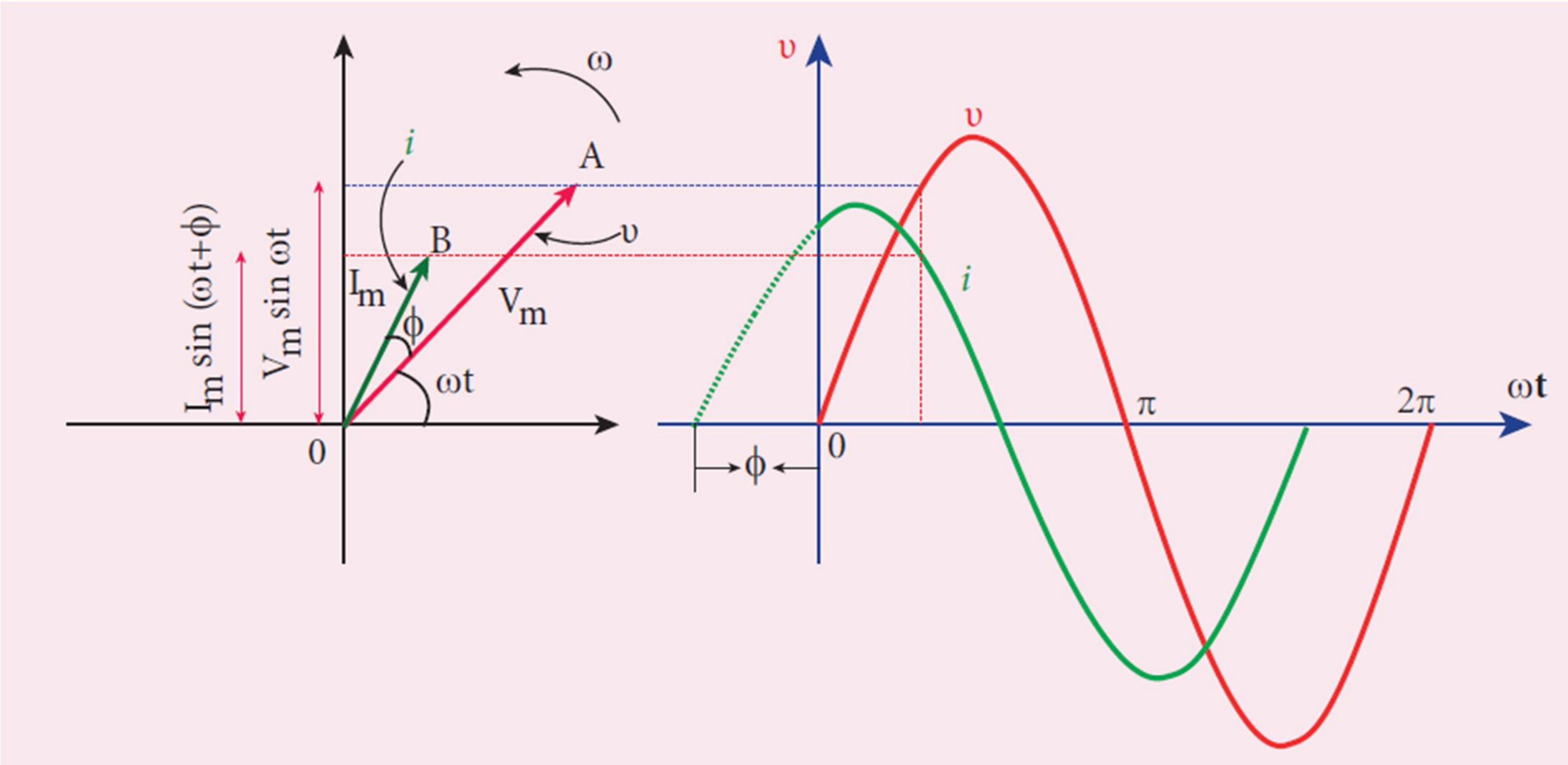


Figure 4.44 Phasor diagram and wave diagram say that *i* leads *υ* by ϕ

4.7.3 AC circuit containing pure resistor

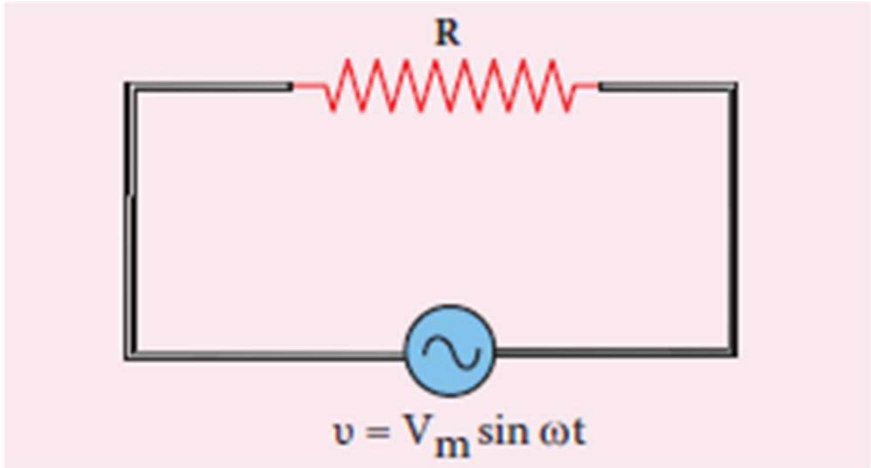


Figure 4.45 AC circuit with resistance

The applied alternating voltage is

$$\upsilon = V_m \sin \omega t$$

The circuit current is

$$i = I_m \sin \omega t$$

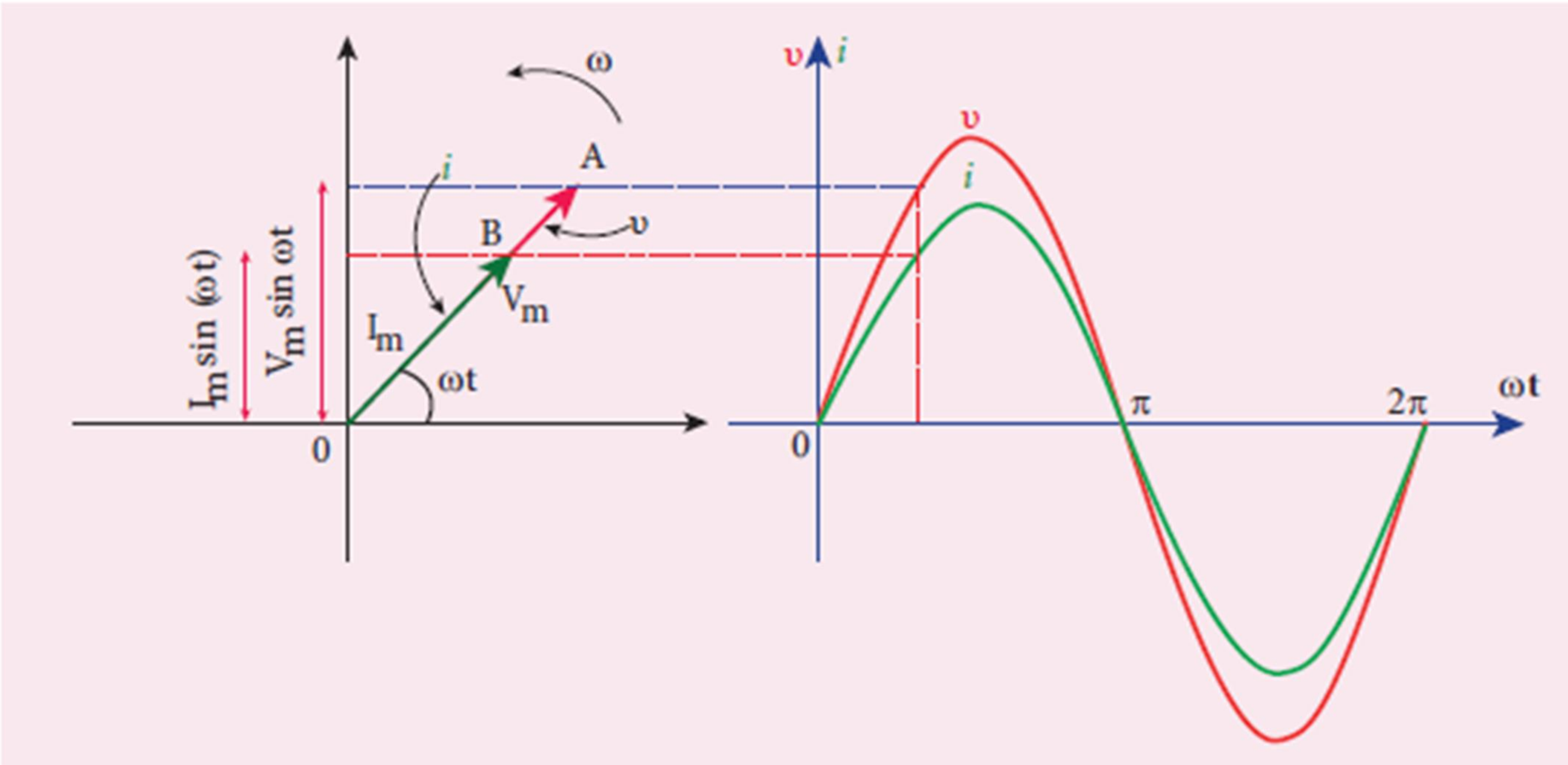
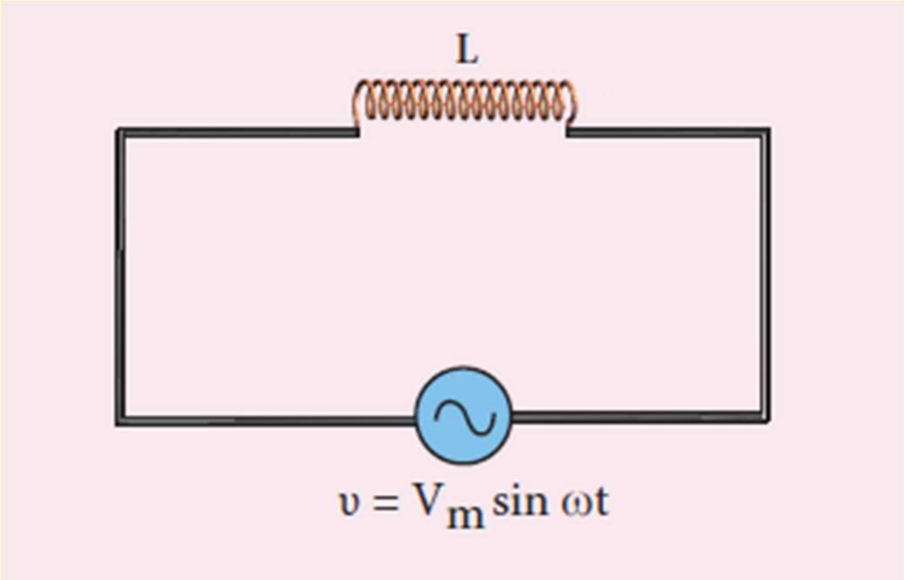


Figure 4.46 Phasor diagram and wave diagram for AC circuit with R

4.7.4 AC circuit containing only an inductor



The applied alternating voltage is

$$\upsilon = V_m \sin \omega t$$

The circuit current is

$$i = I_m \sin\left(\omega t - \pi/2\right)$$

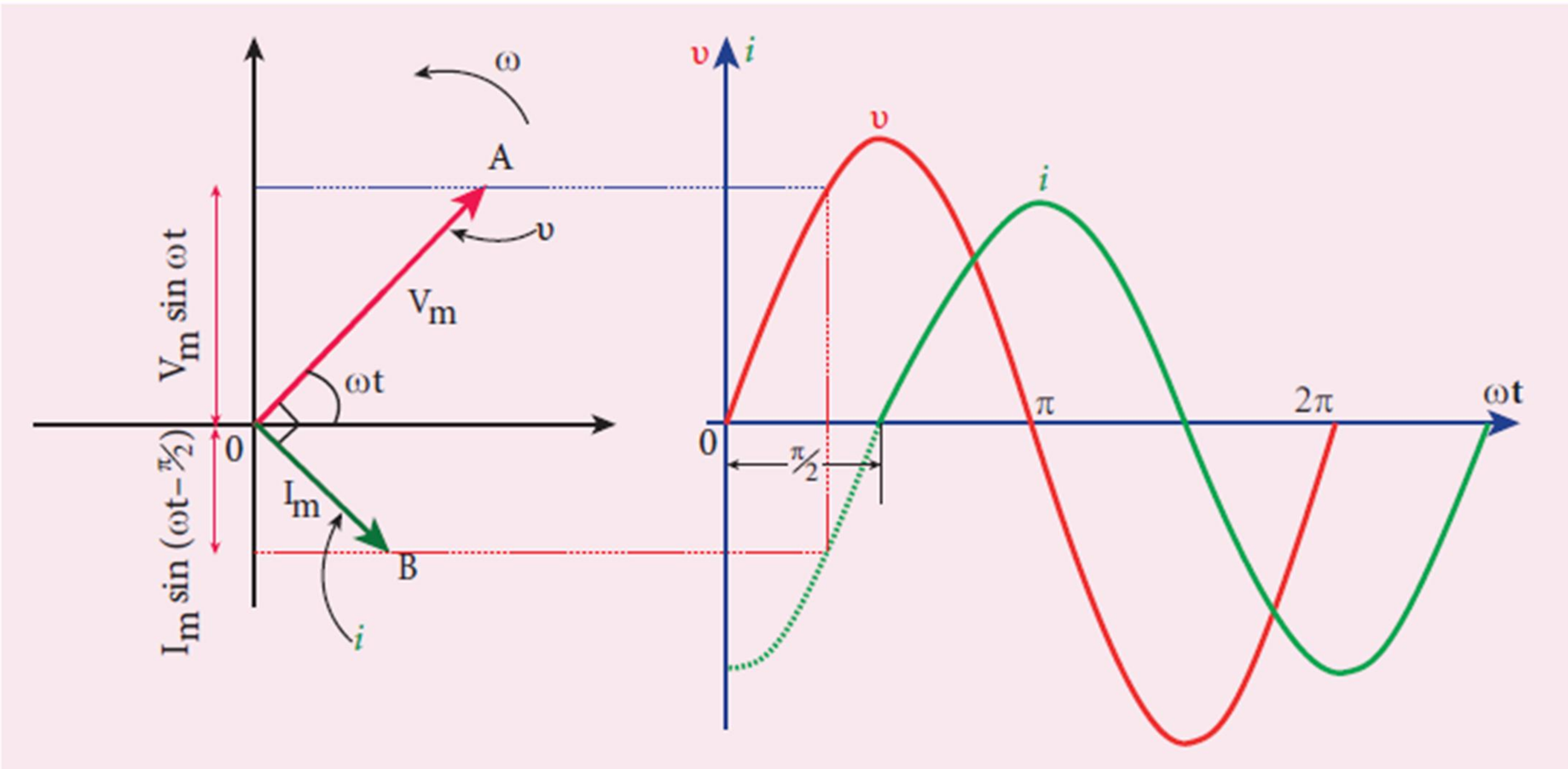
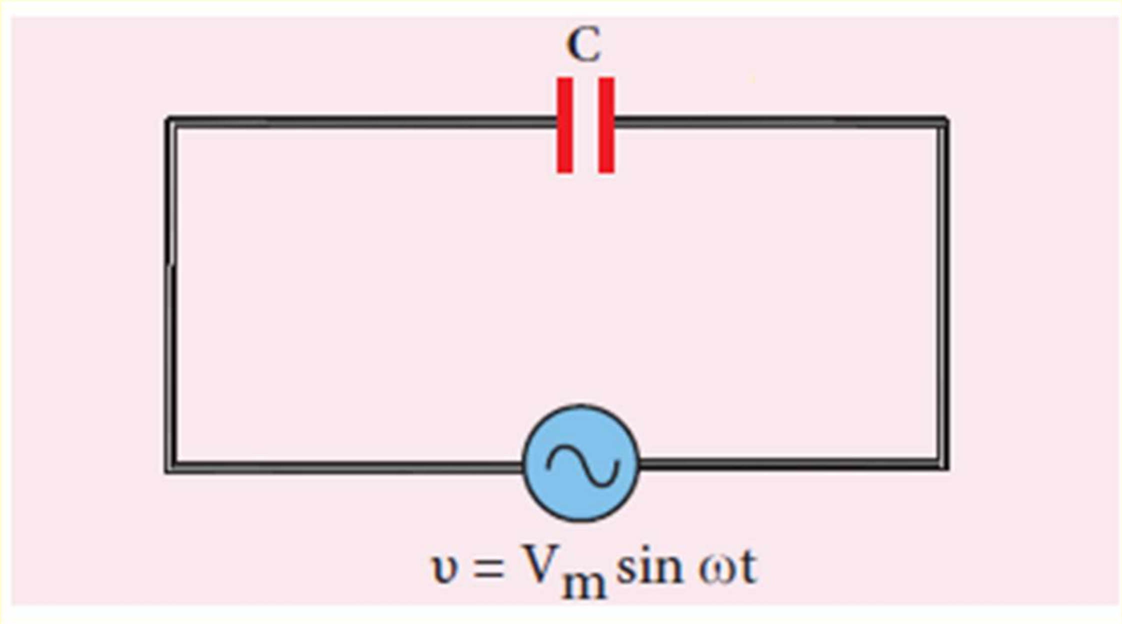


Figure 4.48 Phasor diagram and wave diagram for AC circuit with L

4.7.5 AC circuit containing only a capacitor



The applied alternating voltage is

$$\upsilon = V_m \sin \omega t$$

The circuit current is

$$i = I_m \sin\left(\omega t + \pi/2\right)$$

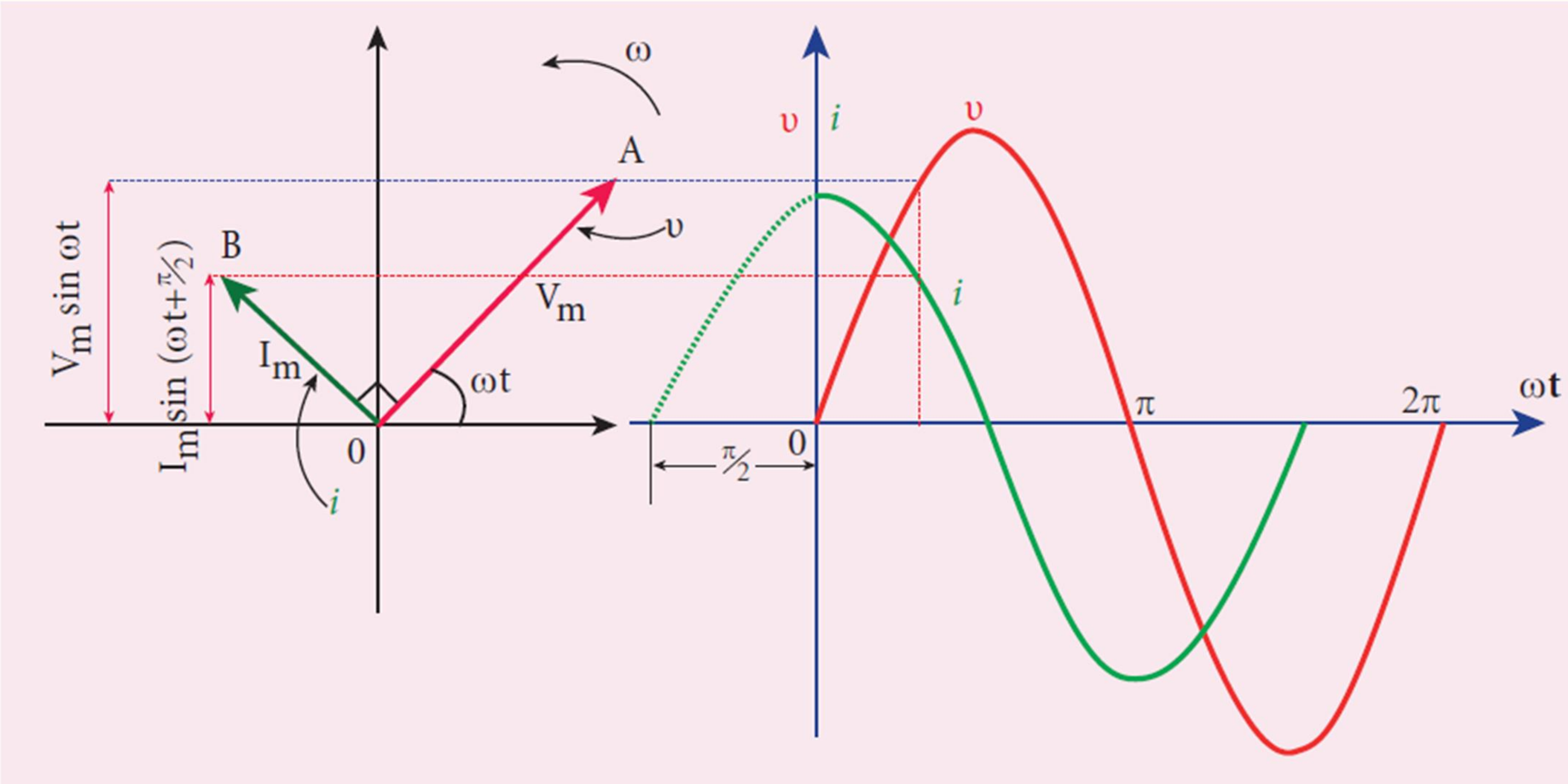
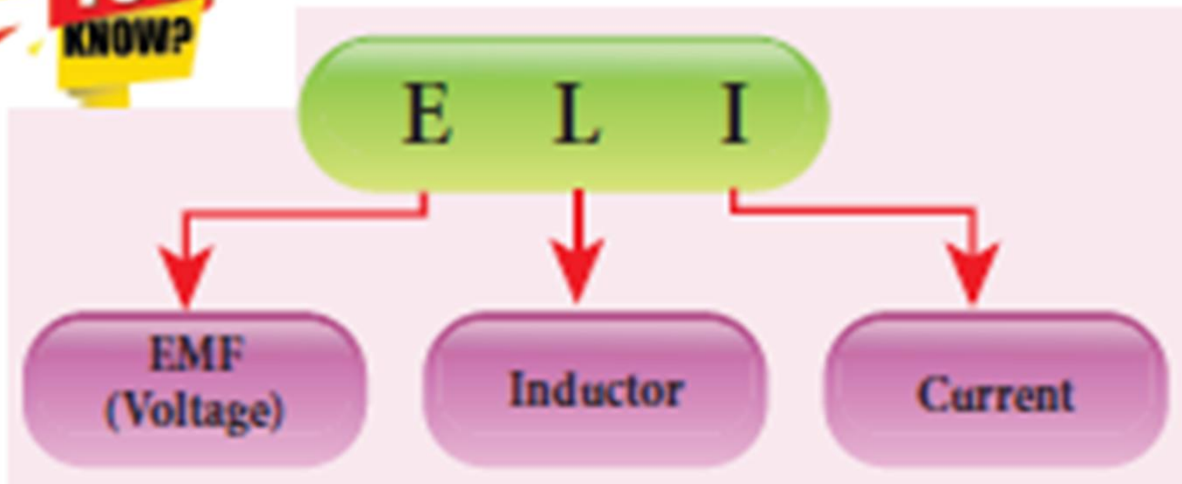


Figure 4.50 Phasor diagram and wave diagram for AC circuit with C

What is ELI?



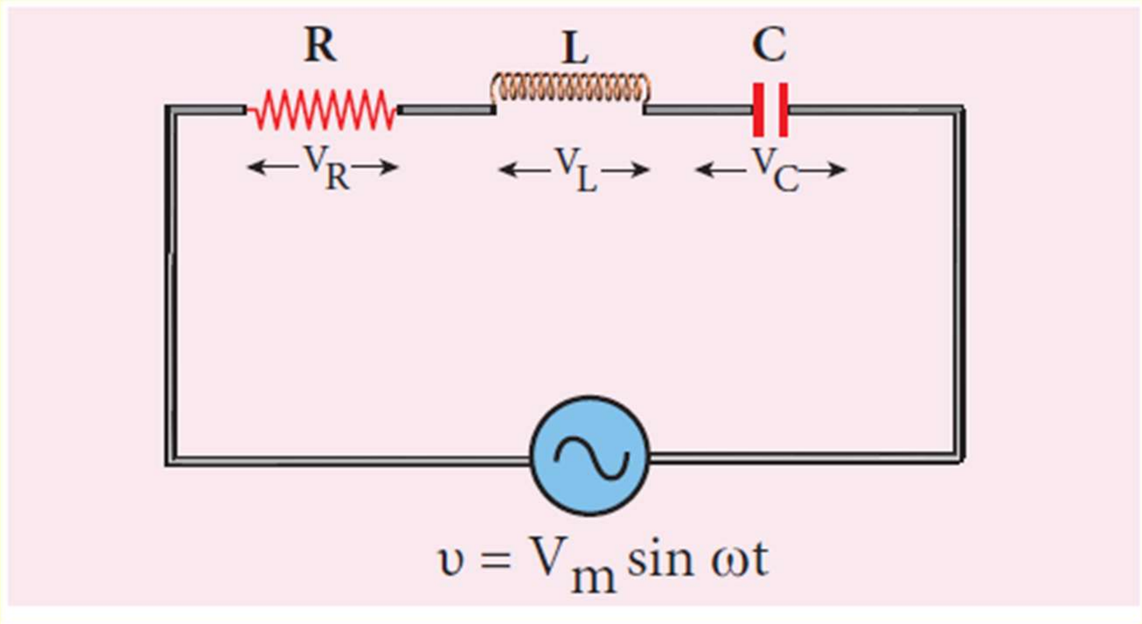
ELI is an acronym which means that EMF (voltage) leads the current in an inductive circuit.

What is ICE?

ICE is an acronym which means that the current leads the EMF (voltage) current in a capacitive circuit.

It is easier to may remember the results of AC circuits with the mnemonic 'ELI the ICE man'.

4.7.6 Series RLC circuit



The applied alternating voltage is

$$\upsilon = V_m \sin \omega t$$

The circuit current is

$$i = I_m \sin(\omega t + \phi)$$

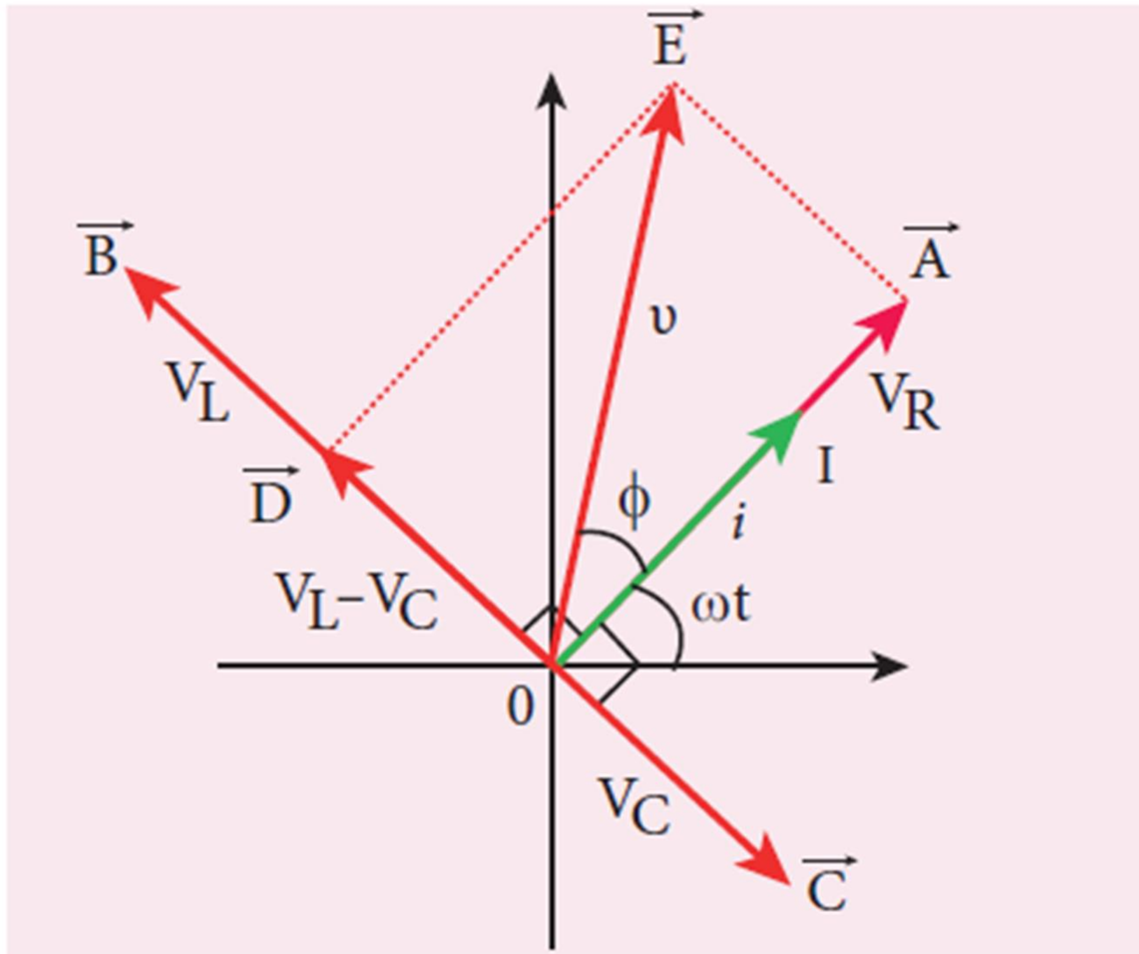


Figure 4.52 Phasor diagram for a series *RLC* – circuit when $V_L > V_C$

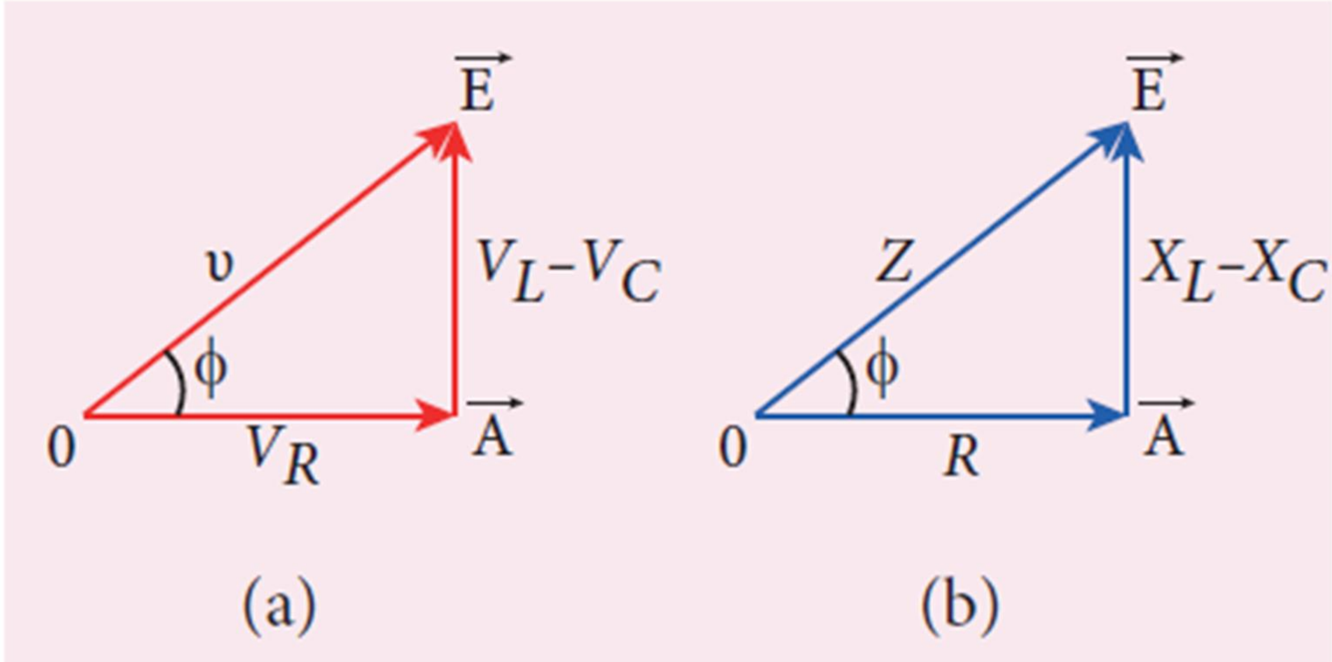


Figure 4.53 Voltage and impedance triangle when $X_L > X_C$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

Table 4.1 Summary of results of AC circuits

Type of Impedance	Value of Impedance	Phase angle of current with voltage	Power factor
Resistance	R	0°	1
Inductance	$X_L = \omega L$	90° lag	0
Capacitance	$X_C = 1/\omega C$	90° lead	0
R- L - C	$\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}$	Between 0° and 90° lag or lead	Between 0 and 1

4.8 Power in AC circuits

4.8.1 Introduction

- the rate of consumption of electric energy in that circuit.
- It is given by the product of the voltage and current.
- In an AC circuit, the voltage and current vary continuously with time.

$$P = v\, i$$

$$= V_m I_m \sin\omega t \sin(\omega t + \phi)$$

$$= V_m I_m \sin\omega t \left[\sin\omega t \cos\phi - \cos\omega t \sin\phi\right]$$

$$P = V_m I_m \left[\cos\phi \sin^2\omega t - \sin\omega t \cos\omega t \sin\phi\right]$$

$$P_{av} = V_{RMS}\ I_{RMS} \cos\phi \quad (4.62)$$

where $V_{RMS} I_{RMS}$ is called apparent power and $\cos\phi$ is power factor. The average power of an AC circuit is also known as the true power of the circuit.

Special Cases

(i) For a purely resistive circuit, the phase angle between voltage and current is zero and $\cos\phi = 1$.

$\therefore P_{av} = V_{RMS}\ I_{RMS}$

(ii) For a purely inductive or capacitive circuit, the phase angle is $\pm \pi/2$ and $\cos(\pm \pi/2) = 0$.

$\therefore P_{av} = 0$

(iii) For series RLC circuit, the phase angle $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$\therefore P_{av} = V_{RMS}\ I_{RMS} \cos\phi$

(iv) For series RLC circuit at resonance, the phase angle is zero and $\cos\phi = 1$.

$\therefore P_{av} = V_{RMS}\ I_{RMS}$

4.8.2 Wattless current

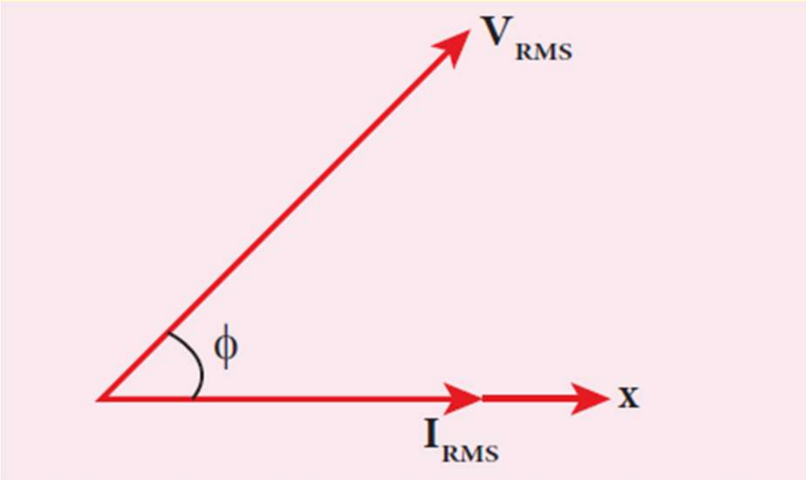


Figure 4.55 V_{RMS} leads I_{RMS} by ϕ

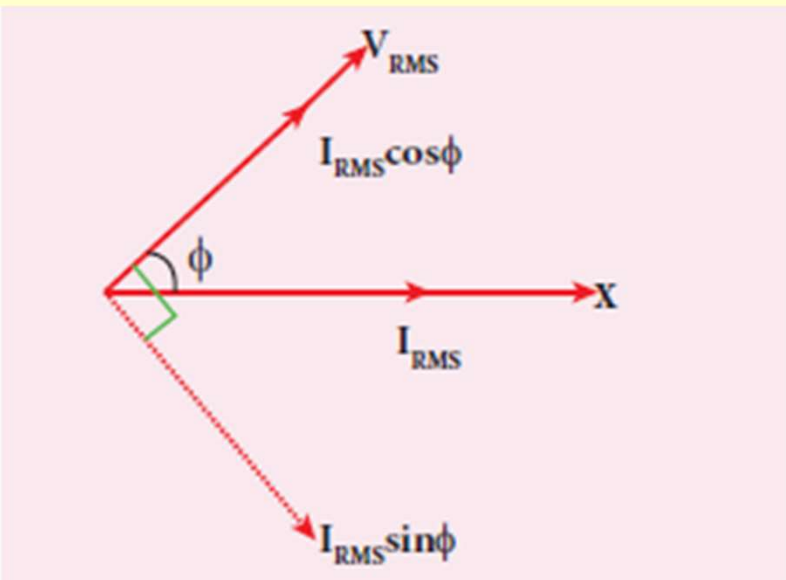


Figure 4.56 The components of I_{RMS}

(i) The component of current $(I_{RMS}\cos\phi)$ which is in phase with the voltage is called active component. The power consumed by this current $= V_{RMS} I_{RMS} \cos\phi$. So that it is also known as 'Wattful' current.

(ii) The other component $(I_{RMS}\sin\phi)$ which has a phase angle of $\pi/2$ with the voltage is called reactive component. The power consumed is zero. So that it is also known as 'Wattless' current.

4.8.3 Power factor

The power factor of a circuit is defined in one of the following ways:

(i) **Power factor = cos ϕ = cosine of the angle of lead or lag**

(ii) **Power factor =** $\dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}}$

(iii) **Power factor =** $\dfrac{VI\cos\phi}{VI}$

$$= \frac{\text{True power}}{\text{Apparent power}}$$

Some examples for power factors:

(i) Power factor = $\cos 0° = 1$ for a pure resistive circuit because the phase angle ϕ between voltage and current is zero.

(ii) Power factor $= \cos\left(\pm \pi/2\right) = 0$ for a purely inductive or capacitive circuit because the phase angle ϕ between voltage and current is $\pm \pi/2$.

(iii) Power factor lies between 0 and 1 for a circuit having R, L and C in varying proportions.

4.9 LC oscillations

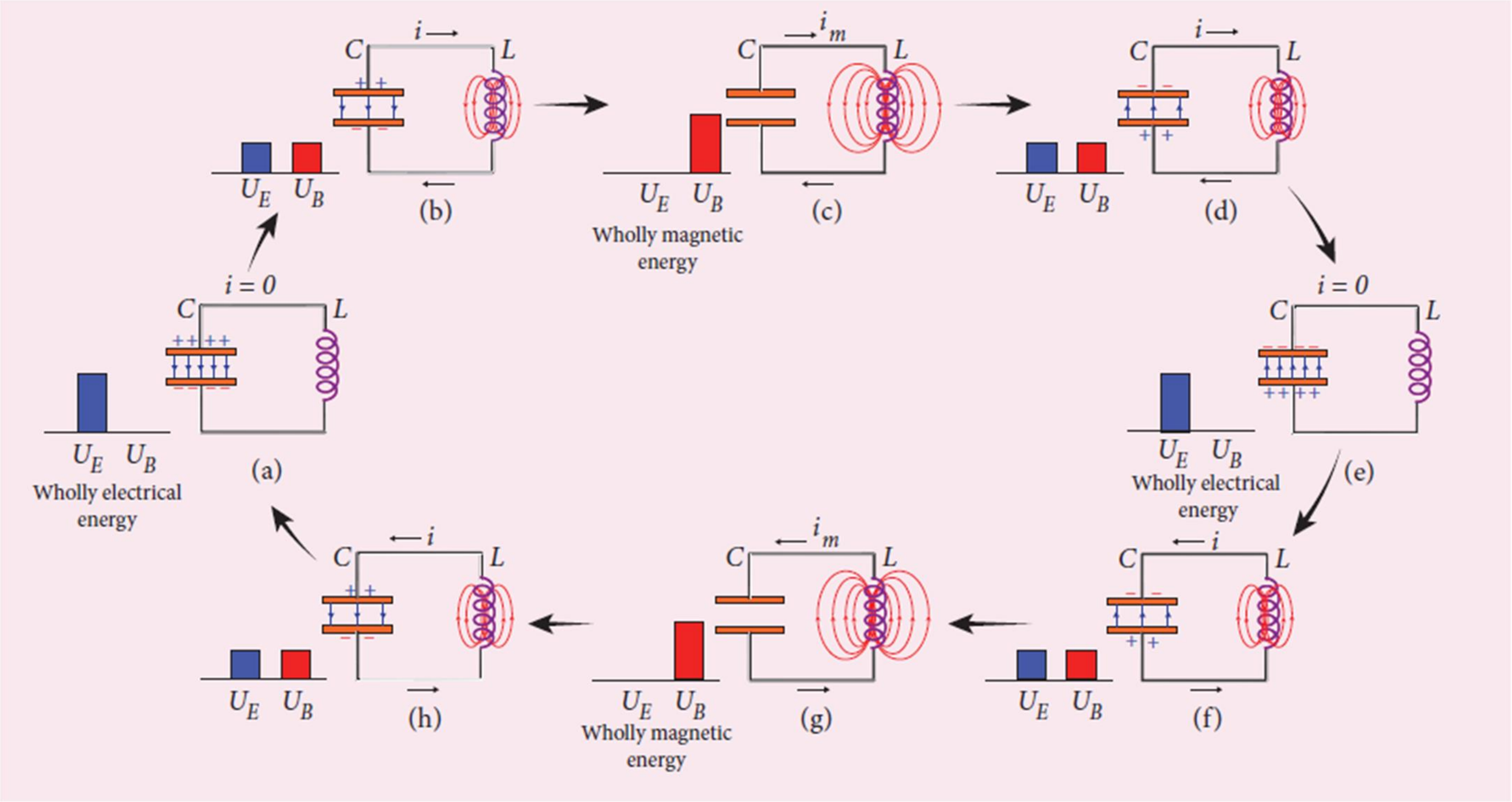


Figure 4.57 LC oscillations

4.9.2 Conservation of energy in LC oscillations

$$\text{Total energy}, U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Let us consider 3 different stages of LC oscillations and calculate the total energy of the system.

Case (i) When the charge in the capacitor, $q = Q_m$ and the current through the inductor, $i = 0$, the total energy is given by

$$U = \frac{Q_m^2}{2C} + 0 = \frac{Q_m^2}{2C} \qquad (4.63)$$

The total energy is wholly electrical.

Case (ii) When charge = 0 ; current = I_m, the total energy is

$$U = 0 + \frac{1}{2}LI_m^2 = \frac{1}{2}LI_m^2$$

$$= \frac{L}{2} \times \left(\frac{Q_m^2}{LC}\right) \quad \text{since } I_m = Q_m\omega = \frac{Q_m}{\sqrt{LC}}$$

$$= \frac{Q_m^2}{2C} \qquad (4.64)$$

The total energy is wholly magnetic.

Case (iii) When charge = q; current = i, the total energy is

$$U = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

Since $q = Q_m \cos\omega t$, $i = -\frac{dq}{dt} = Q_m\omega \sin\omega t$.

The negative sign in current indicates that the charge in the capacitor decreases with time.

$$U = \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{L\omega^2 Q_m^2 \sin^2 \omega t}{2}$$

$$= \frac{Q_m^2 \cos^2 \omega t}{2C} + \frac{L Q_m^2 \sin^2 \omega t}{2LC} \quad \text{since } \omega^2 = \frac{1}{LC}$$

$$= \frac{Q_m^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right)$$

$$U = \frac{Q_m^2}{2C} \tag{4.65}$$

From above three cases, it is clear that the total energy of the system remains constant.

4.9.3 Analogies between LC oscillations and simple harmonic oscillations

Qualitative Treatment:

Table 4.3 Energy in two oscillatory systems

LC oscillator		Spring-mass system	
Element	**Energy**	**Element**	**Energy**
Capacitor	Electrical Energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Spring	Potential energy $= \frac{1}{2}k\,x^2$
Inductor	Magnetic energy $= \frac{1}{2}Li^2 \quad i = \frac{dq}{dt}$	Mass	Kinetic energy $= \frac{1}{2}mv^2 \quad v = \frac{dx}{dt}$

Analogies between LC oscillations and simple harmonic oscillations

Table 4.4 Analogies between electrical and mechanical quantities

Electrical system	Mechanical system
Charge q	Displacement x
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $= \frac{1}{2}k x^2$
Magnetic energy $= \frac{1}{2}Li^2$	Kinetic energy $= \frac{1}{2}mv^2$
Electromagnetic energy $U = \frac{1}{2}\left(\frac{1}{C}\right)q^2 + \frac{1}{2}Li^2$	Mechanical energy $E = \frac{1}{2}k x^2 + \frac{1}{2}mv^2$

Quantitative Treatment:

The electromagnetic energy of the LC system is given by

$$U = \frac{1}{2} L i^2 + \frac{1}{2}\left(\frac{1}{C}\right) q^2 = \text{constant}$$

Differentiating U with respect to time,

$$\frac{dU}{dt} = \frac{1}{2} L \left(2i \frac{di}{dt}\right) + \frac{1}{2C}\left(2q \frac{dq}{dt}\right) = 0$$

$$\boxed{L\frac{d^2 q}{dt^2} + \frac{1}{C} q = 0} \qquad \because \; i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2 q}{dt^2}$$

The general solution of equation is of the form

$$q(t) = Q_m \cos(\omega t + \phi)$$

Current in the circuit:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}\left[q_m \cos(\omega t + \phi)\right]$$

$$= -Q_m \omega \sin(\omega t + \phi)$$

$$i(t) = -I_m \sin(\omega t + \phi) \qquad \because \; I_m = Q_m \omega$$

Angular frequency:

$$\frac{d^2 q}{dt} = -Q_m \omega^2 \cos(\omega t + \phi)$$

$$L\left[-Q_m \omega^2 \cos(\omega t + \phi)\right] + \frac{1}{C} Q_m \cos(\omega t + \phi) = 0$$

$$\omega = \frac{1}{\sqrt{LC}}$$

Oscillations of electrical and magnetic energy:

The electrical energy of the LC system is

$$U_E = \frac{q^2}{2C} = \frac{Q_m^2}{2C}\cos^2\left(\omega t + \phi\right)$$

The magnetic energy is

$$U_B = \frac{1}{2}Li^2 = \frac{L}{2}I_m^2\sin^2\left(\omega t + \phi\right)$$

$$= \frac{L}{2}\left(Q_m^2\omega^2\right)\sin^2\left(\omega t + \phi\right) \qquad \because \omega = \frac{1}{\sqrt{LC}}$$

$$U_B = \frac{Q_m^2}{2C}\sin^2\left(\omega t + \phi\right)$$

Total energy is

$$U = U_E + U_B = \frac{Q_m^2}{2C}$$

Oscillations of electrical and magnetic energy:

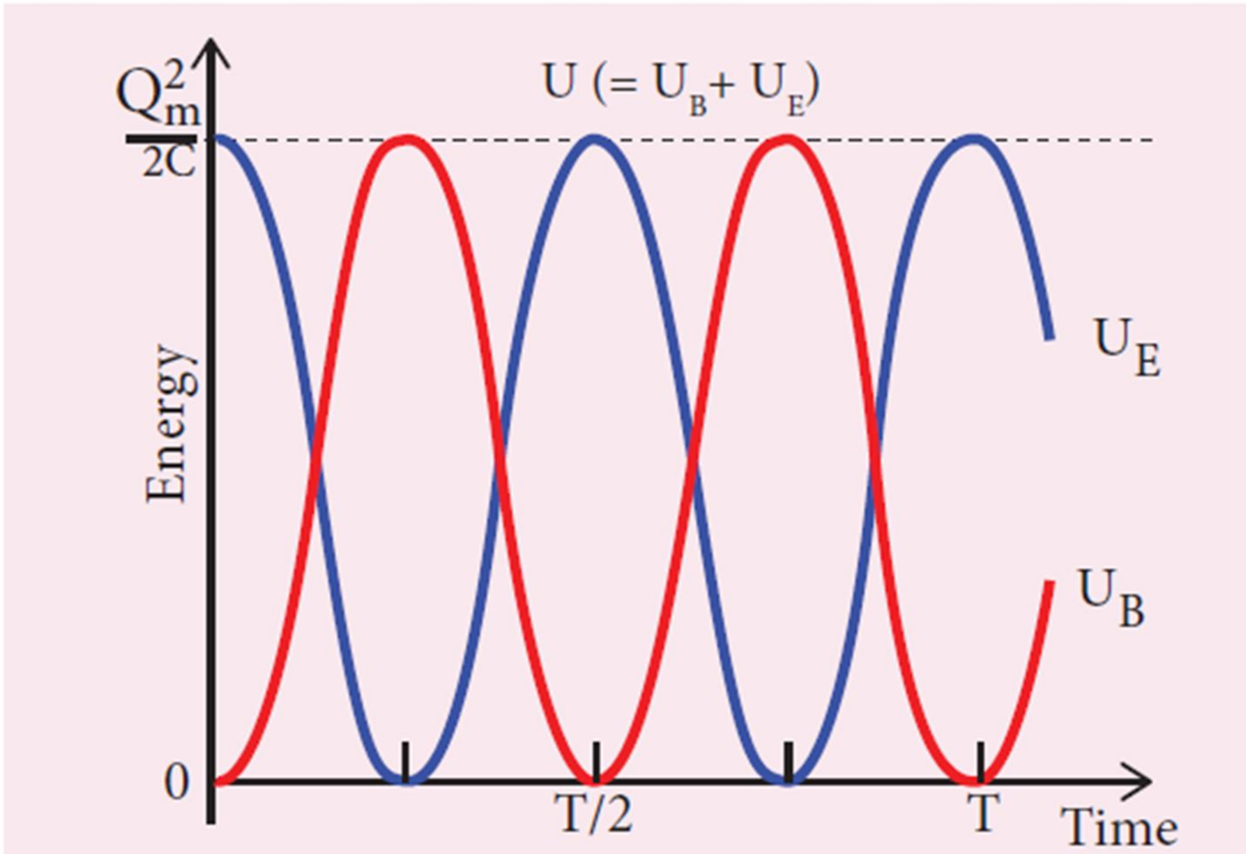
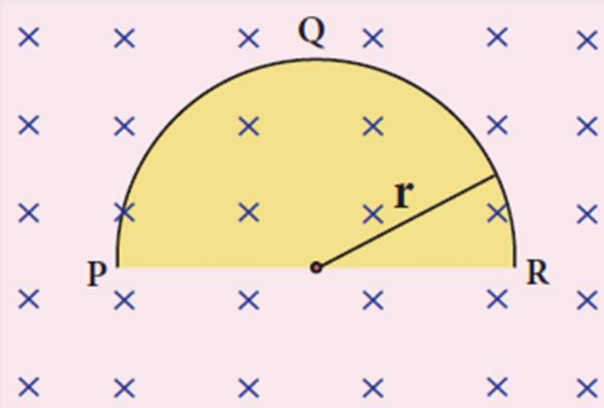


Figure 4.58 The variation of U_E and U_B as a function of time

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure.



The potential difference developed across the ring when its speed v, is

(NEET 2014)

(a) Zero

(b) $\frac{Bv\pi r^2}{2}$ and P is at higher potential

(c) πrBv and R is at higher potential

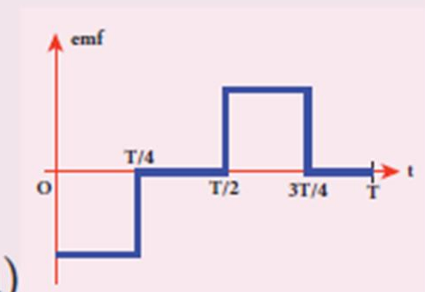
(d) $2rBv$ and R is at higher potential

(2) $\varepsilon = Bl v$

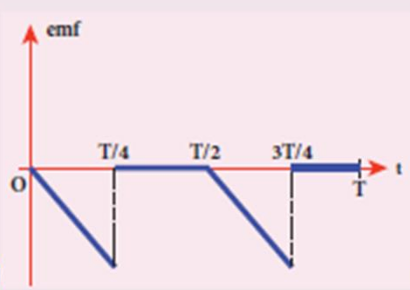
Since $l = 2r$, the effective length between P and R

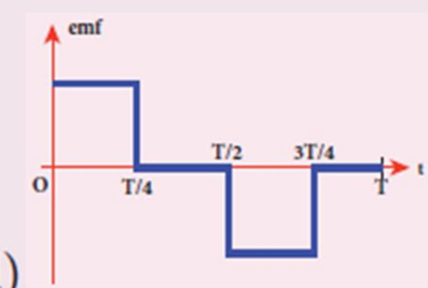
$$\boxed{\begin{array}{l}\varepsilon = 2rBv \\ \text{R is at higher pot.}\end{array}}$$

5. The current i flowing in a coil varies with time as shown in the figure. The variation of induced emf with time would be (NEET – 2011)

(a)


(b)


(c)


(d)


5

$$\varepsilon = -L\frac{di}{dt}$$

or

$$\varepsilon \propto -\frac{di}{dt}$$

9. In an electrical circuit, R, L, C and AC voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage and current in the circuit is $\pi/3$. Instead, if C is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is

(NEET 2012)

(a) $1/2$ (b) $1/\sqrt{2}$

(c) 1 (d) $\sqrt{3}/2$

9) when L is removed,

$$\tan\phi_1 = \frac{X_C}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_C}{R}$$

$$\Rightarrow X_C = \sqrt{3}R$$

when C is removed,

$$\tan\phi_2 = \frac{X_L}{R}$$

$$\Rightarrow X_L = \sqrt{3}R$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

Power factor $= \cos\phi$

$$= \frac{R}{Z} = \frac{R}{R}$$

$$\boxed{PF = 1}$$

11. In a series resonant *RLC* circuit, the voltage across 100 Ω resistor is 40 V. The resonant frequency ω is 250 rad/s. If the value of C is 4 μF, then the voltage across L is

(a) 600 V (b) 4000 V

(c) 400V (d) 1 V

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$V_L = I X_L$

$I = \frac{V_R}{R}$

$X_L = X_C = \frac{1}{\omega C}$

12. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf v = 10 sin 340 t. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W

(c) 0.46 W (d) 0.67 W

(12)

$$\boxed{P = V_{Rms} I_{Rms} \cos\phi}$$

$$V_{Rms} = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

$$I_{Rms} = \frac{I_m}{\sqrt{2}} \quad \left[\because I_m = \frac{V_m}{Z}\right]$$

$$= 10/65.62$$

$$X_L = L\omega$$

$$= 6.8\ \Omega$$

$$X_C = 1/C\omega = 58.82\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 65.62\ \Omega$$

$$P = \frac{10}{\sqrt{2}} \times \frac{10}{65.62 \times \sqrt{2}} \times \frac{40}{65.62}$$

$$\left[\because \cos\phi = \frac{R}{Z}\right]$$

$$\boxed{P = 0.46\ W}$$

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$

(c) $\frac{Q}{\sqrt{2}}$ (d) Q

14 $(U_E)_m = \frac{Q^2}{2C}$

$$U_E = \frac{(U_E)_m}{2} = \frac{q^2}{2C}$$

$$\Rightarrow \frac{q^2}{2C} = \frac{1}{2}\left(\frac{Q^2}{2C}\right)$$

$$\boxed{q = \frac{Q}{\sqrt{2}}}$$

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180°. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

(Ans: 48×10^{-3} C)

Ex 5

$\Delta\phi = -2BA$

$\Rightarrow \varepsilon \Delta t = 2NBA$

$q = i\Delta t$

$= \frac{\varepsilon \Delta t}{R}$

$\boxed{q = 48\, mC}$

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm^2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

14. $B_1 = \mu_0 n_1 i_1 = 10.04 \times 10^{-2}\ wb/m^2$

$(\Phi_{21})_i = B_1 A_2 \cos\theta \quad \theta = 0^\circ$

$= 4.02 \times 10^{-5}\ wb$

$(\Phi_{21})_f = -4.02 \times 10^{-5}\ wb$

$(\because \theta = 180^\circ)$

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

$$\boxed{\mathcal{E}_2 = 0.20\ V}$$

15. A 200 turn coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil.

(Ans: 2.84 mH)

15 $M_{coil} = \mu_0 n_1 n_2 A_2 l_2$

$= \mu_0 n_1 N_2 A_2$ $\left(\because n_2 l_2 = N_2\right)$

$M_{coil} = 2.84\,mH$

19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

(Ans: 8.2 kW and 2.48 kW)

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$$V_p = \frac{V_s N_p}{N_s} = 400V$$

$$I_s = \frac{P_o}{V_s} = 20 A$$

$$P_i = \frac{P_o}{\eta} = 40 kW$$

$$I_p = \frac{P_i}{V_p} = 100 A$$

$$P_p = I_p^2 R_p = 8.2 kW$$

$$P_s = I_s^2 R_s = 2.48 kW$$

Thank you !!!